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Problem 31.39 (RHK)

We have to calculate (a) *the energy density of the electric field at a distance r from an electron* (*presumed to be a particle*) *at rest.* (b) *We will assume that the electron is not a point but a sphere of radius R over whose surface the electron charge is uniformly distributed. We will determine the energy associated with the external electric field in vacuum of the electron as a function of R.* (c) *We will now associate this energy with the mass of the electron, using* $E_0 = mc^2$*, and calculate the value for R. We will evaluate this radius numerically*; *it is often called the classical radius of the electron.*

Solution:

Assuming that electron is a charged sphere of radius R over whose surface charge e is uniformly distributed, electric field for $r \geq R$ will be

$$E(r) = \frac{e}{4\pi\varepsilon_0 r^2} \; .$$

Therefore, energy density of the electric field in the space outside the electron will be

$$u(r) = \frac{1}{2}\varepsilon_0 E^2(r) = \frac{1}{2}\varepsilon_0\left(\frac{e}{4\pi\varepsilon_0 r^2}\right)^2 = \frac{e^2}{32\pi^2\varepsilon_0 r^4}.$$

Therefore, the total energy associated with an electron of radius *R* and charge *e* will be

$$\mathrm{E} = \int_R^\infty 4\pi r^2 dr u(r) = \int_R^\infty 4\pi r^2 dr \times \frac{e^2}{32\pi^2\varepsilon_0 r^4} = \frac{e^2}{8\pi\varepsilon_0}\int_R^\infty \frac{dr}{r^2}$$

$$= \frac{e^2}{8\pi\varepsilon_0 R}.$$

We now associate this energy with the mass of the electron. That is

$$\mathrm{E} = \frac{e^2}{8\pi\varepsilon_0 R} = mc^2.$$

Or

$$R = \frac{e^2}{8\pi\varepsilon_0 mc^2},$$

which is called the *classical radius* of the electron.

We next calculate its numerical value

$$R = \frac{\left(1.6\times10^{-19}\right)^2\times8.99\times10^9}{2\times9.11\times10^{-31}\times\left(3\times10^8\right)^2}\ \mathrm{m}$$

$$= 0.140\times10^{-14}\ \mathrm{m} = 1.4\times10^{-15}\ \mathrm{m} = 1.4\ \mathrm{fm}.$$