370.

Problem 31.38 (RHK)

A cylindrical capacitor has radii a and b. we have to show that half the stored electrical potential energy lies within a cylinder whose radius is

$$r = \sqrt{ab} \ .$$

Solution:

Let the length of the cylindrical capacitor with inner radius *a* and outer radius *b* be *L*. Electric field at a distance r from the axis of the cylinder when the charge on the capacitor is *q* can be found by applying Gauss' law.

We have

$$\varepsilon_0 \left(2\pi r L\right) E\left(r\right) = q,$$

or

$$E\left(r\right) = \frac{q}{2\varepsilon_0 \pi r L} \ .$$

We will use the fact that in a long cylindrical capacitor electric potential energy lies within the cylindrical space as the electric field is zero in the space outside the capacitor. Also, the volume density of energy due to electric field is

$$u = \frac{1}{2}\varepsilon_0 E^2.$$

Therefore, the total energy due to electric field in the cylindrical capacitor will be

$$U = \frac{1}{2}\varepsilon_0 \int_a^b (2\pi r L) dr E^2(r).$$

Let half the energy due to electric field that is $\frac{U}{2}$ be contained within the annular space of radius *R*. We, therefore, have

$$\frac{1}{2}\int_a^R \varepsilon_0 (2\pi r L) dr E^2(r) = \frac{1}{4}\varepsilon_0 \int_a^b (2\pi r L) dr E^2(r),$$

or

$$\int_a^R \frac{dr}{r} = \frac{1}{2}\int_a^b \frac{dr}{r}.$$

This gives

$$\ln\left(\frac{R}{a}\right) = \frac{1}{2}\ln\left(\frac{b}{a}\right),$$

or

$$\ln\left(\frac{R^2}{ab}\right) = 0.$$

Or

$$\frac{R^2}{ab} = 1,$$

and

$$R = \sqrt{ab} \ .$$