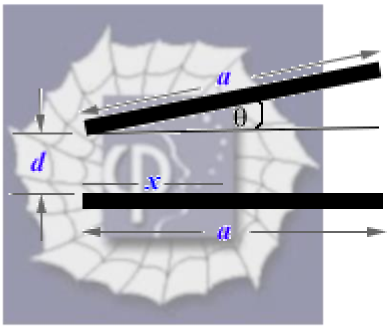
367.

Problem 31.26 (RHK)

A capacitor has square plates, each of side *a*, making an angle θ with each other as shown in the figure. We have to show that for small θ the capacitance is given by

$$C = \frac{\varepsilon_0 a^2}{d}\left(1 - \frac{a\theta}{2d}\right).$$



Solution:

A capacitor has square plates, each of side a, making an angle θ with each other as shown in the figure. For calculating the capacitance of this capacitor, we will assume that it is a parallel combination of strip-capacitances along the length *a* each of area

$$A = a\,dx\ .$$

The separation of the strip-capacitance at length *x* from the end as shown in the figure will be

$$d(x) = d + x\theta.$$

The capacitance of the strip-capacitor at length *x* will be

$$c(x) = \frac{\varepsilon_0 a dx}{d(x)} = \frac{\varepsilon_0 a\, dx}{d + x\theta}.$$

Therefore, the capacitance of the capacitor shown in the figure will be

$$C = \int_0^a c(x) = \varepsilon_0 a \int_0^a \frac{dx}{d + x\theta} = \frac{\varepsilon_0 a}{d} \int_0^a dx \left(1 + \frac{x\theta}{d}\right)^{-1}.$$

In the approximation

$$\frac{x\theta}{d} = 1,$$

integral reduces to

$$C = \frac{\varepsilon_0 a}{d} \int_0^a dx \left(1 - \frac{x\theta}{d}\right) = \varepsilon_0 a^2 \left(1 - \frac{a\theta}{2d}\right).$$