360.

Problem 30.64 (RHK)

We have to estimate (a) *the charge required to raise an isolated metallic sphere of radius* 1.0-m *radius to a potential of* 1.0 MV. (b) *We have to estimate the charge required to raise an isolated metallic sphere of radius* 1.0-cm *to the same amount of potential,* 1.0 MV. (c) *We have to answer why in an electrostatic accelerator a large metal sphere is deployed when the same potential can be achieved using a smaller charge with a small sphere.*

Solution:

Charge required on the surface of a metal sphere of radius R for it to be at potential V is given by the relation

$$Q = 4\pi\varepsilon_0 RV = \frac{RV}{8.99\times10^9} \text{ C.}$$

(a)

For a metal sphere of radius of 1.0 m to be at a potential of 1.0×10^{6} V amount of charge required will be

$$Q_1 = \frac{10^6}{8.99\times10^9}\ \text{C} = 111\ \mu\text{C}.$$

(b)

For a metal sphere of radius of 1.0 cm to be at a potential of 1.0×10^{6} V amount of charge required will be

$$Q_{0.01} = 1.11\ \mu\text{C}.$$

(c)

Electric field near a charged metal surface is proportional to its surface charge density.

The surface charge density for a 1.0-m metal sphere to be at 1 MV potential, will be

$$\sigma_1 = \frac{111}{4\pi}\ \mu\text{C m}^{-2} = 8.83\ \mu\text{C m}^{-2}.$$

And the surface charge density for a 1.0-cm metal sphere to be at 1 MV potential, will be

$$\sigma_{0.01} = 8.83\times10^{2}\ \mu\text{C m}^{-2}.$$

Therefore, in order to have not too large electric field near the surface of charged conductor, it is preferred to deploy in electrostatic accelerators a metal sphere of a large radius.