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<u>Problem 30.48 (RHK)</u>

Rutherford gave the following expression for the electric potential inside an atom containing a point positive charge Ze at its centre and surrounded by a distribution of negative electricity, –Ze uniformly distributed within a sphere of radius R:

$$V(r)=\frac{Ze}{4\pi\varepsilon_0}\left(\frac{1}{r}-\frac{3}{2R}+\frac{r^2}{2R^3}\right).$$

(a) *We have to show that it leads to the expression for the electric field given by*

$$E(r)=\frac{Ze}{4\pi\varepsilon_0}\left(\frac{1}{r^2}-\frac{r}{R^3}\right).$$

We have to answer why this expression does not go to zero as $r\to\infty$.

<u>Solution:</u>

(a)

Rutherford's expression for potential inside an atom is

$$V(r)=\frac{Ze}{4\pi\varepsilon_0}\left(\frac{1}{r}-\frac{3}{2R}+\frac{r^2}{R^3}\right).$$

Relation between electric field E and potential $V(r)$ is

$$E(r) = -\frac{dV(r)}{dr}.$$

We compute E by differentiating the function $V(r)$ and note that

$$-\frac{dV(r)}{dr} = \frac{d}{dr}\left(\frac{Ze}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{3}{2R} + \frac{r^2}{2R^3}\right)\right)$$

$$= \frac{Ze}{4\pi\varepsilon_0}\left(\frac{1}{r^2} - \frac{r}{R^3}\right)$$

$$= E(r)\ .$$

(b)

We note that in Rutherford's function for $V(r)$

$$V(R) = 0.$$

That is zero of the potential is chosen at the surface of the sphere of radius R, the net charge within which is zero. Therefore, this expression does not go to zero as $r \to \infty$.