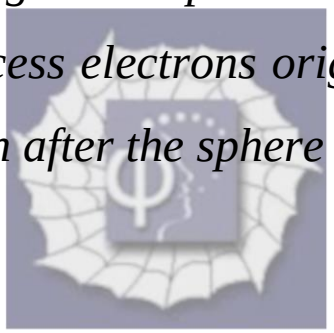
355.

Problem 30.57 (RHK)

We consider the Earth to be a spherical conductor of radius 6370 *km and to be initially uncharged. A metal sphere, having a radius* 13 cm *and carrying a charge* –6.2 nC *is earthed, that is, put into electrical contact with the Earth. We have to show that this process effectively discharges the sphere. We will calculate the fraction of the excess electrons originally present on the sphere that remain after the sphere is earthed.*



Solution:

Let the charge on the after it has been connected to a charged conducting sphere of radius 13 cm and charge –6.2 nC be *Q*. If the Earth is considered to be a conducting sphere, then after the conducting sphere has been earthed, the Earth and the sphere will become equipotential. That is we have the following condition on the distributed charge

$$\frac{Q}{4\pi\varepsilon_0 R} = \frac{q-Q}{4\pi\varepsilon_0 r},$$

where radius of the Earth is

$R = 6370 \times 10^3$ m,

and radius of the conducting sphere is

$r = 13.0 \times 10^{-2}$ m.

Therefore,

$Qr = qR - QR,$

and

$$Q = \frac{qR}{R + r}.$$

Charge on the conducting sphere after it has been earthed will be

$$q - Q = q - \frac{qR}{(R + r)} = \frac{qr}{R\left(1 + \frac{r}{R}\right)}; \ \frac{qr}{R}, \text{ as } \frac{r}{R} = 1.$$

Therefore,

$$q - Q = -\frac{6.2 \times 10^{-19} \times 13 \times 10^{-2}}{6.370 \times 10^6} \text{ C} = 1.265 \times 10^{-17} \text{ C}.$$

The charge left on the conducting sphere is about that of

$\frac{1.265 \times 10^{-17}}{1.6 \times 10^{-19}}$; 79 electrons. And it is effectively discharged.

The fraction of the electrons that remain on the conducting sphere after it has been earthed will be

$$\frac{q-Q}{q}=\frac{r}{R}=\frac{13\times10^{-2}}{6.370\times10^{6}}=2.0\times10^{-8}.$$