

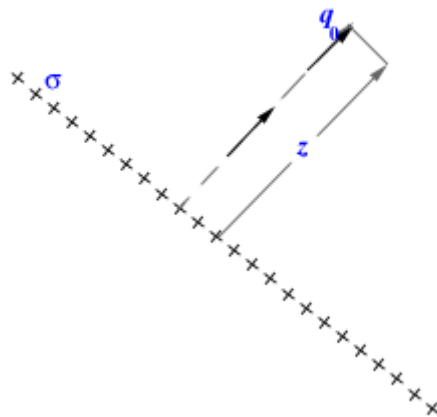
351.

Problem 30.36 (RHK)

In the figure an “infinite” sheet of positive charge density σ has been shown. (a) We have to calculate the work done by the electric field of the sheet as a small positive test charge q_0 is move from an initial position on the sheet to a final position located a perpendicular distance z from the sheet. (b) We have to use the result from (a) to show that the electric potential of an infinite sheet of charge can be written

$$V = V_0 - (\sigma/2\epsilon_0)z,$$

where V_0 is the potential at the surface of the sheet.



Solution:

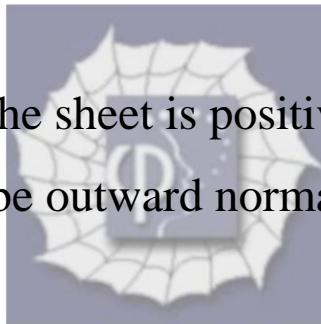
As we are considering an “infinite” nonconducting sheet electric field will be perpendicular to the plane of the sheet and its magnitude will be independent of the distance z from the sheet. By applying Gauss’ law, we have

$$\varepsilon_0 2AE = A\sigma,$$

or

$$E = \frac{\sigma}{2\varepsilon_0}.$$

As the charge on the sheet is positive, the direction of the electric field will be outward normal to the plane of the sheet.



(a)

Force on a test charge q_0 will be

$$\vec{F} = q_0 \vec{E} = \frac{q_0 \sigma \hat{z}}{2\varepsilon_0}.$$

Therefore, the work done by the field on moving charge q_0 from the surface of the sheet a distance z will be

$$W = \int_0^z \vec{F} \cdot d\hat{z} = \int_0^z \frac{q_0 \sigma}{2\varepsilon_0} dz = \frac{q_0 \sigma z}{2\varepsilon_0}.$$

(b)

Definition of electric potential is

$$V = V_0 - \frac{W}{q_0} = V_0 - \frac{\sigma z}{2\epsilon_0}.$$

