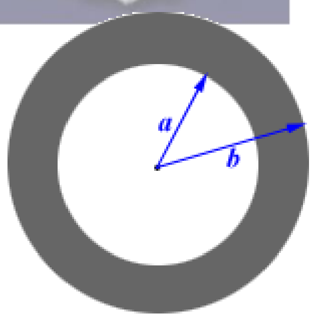
350.

<u>Problem 30.38 (RHK)</u>

A total amount of positive charge Q is spread onto a nonconducting flat circular annulus of inner radius a and outer radius b. The charge is so distributed so that the charge density (charge per unit area) is given by $\sigma = k/r^3$, *where r is the distance from the centre of the annulus to any point on it. We have to show that the potential at the centre of the annulus is given by*

$$V = \frac{Q}{8\pi\varepsilon_0}\left(\frac{a+b}{ab}\right).$$



<u>Solution:</u>

Charge density inside the nonconducting flat circular annulus is

$$\sigma = k/r^3 .$$

The total charge in the annulus is Q. Therefore, we have

$$Q=\int_a^b 2\pi r\sigma(r)dr=\int_a^b 2\pi r\frac{k}{r^3}dr=-2\pi k\left[\frac{1}{r}\right]_a^b=-2\pi k\left(\frac{1}{b}-\frac{1}{a}\right)$$

$$=2\pi k\frac{(b-a)}{ab}.$$

And

$$k=\frac{Qab}{2\pi(b-a)}\ .$$

By considering a ring of radius r concentric with the annulus and inside the annulus, and using the definition of electric potential we calculate the potential at the centre by integration

$$V=\int_a^b\frac{2\pi r\sigma dr}{4\pi\varepsilon_0 r}=\int_a^b\frac{2\pi rkdr}{4\pi\varepsilon_0 r^4}=-\frac{k}{4\varepsilon_0}\left[\frac{1}{r^2}\right]_a^b=\frac{k}{4\varepsilon_0}\left(\frac{1}{a^2}-\frac{1}{b^2}\right)$$

$$=\frac{k}{4\varepsilon_0}\frac{\left(b^2-a^2\right)}{a^2b^2}\ .$$

Substituting for k in V, we get

$$V=\frac{Q}{8\pi\varepsilon_0}\left(\frac{a+b}{ab}\right)\ .$$