341.

Problem 30.13 (RHK)

A particle of (positive) charge Q is assumed to have a fixed position at P. A second particle of mass m and (negative) charge $-q$ *moves at a constant speed in a circle of radius* r_1*, centred at P. We have to derive an expression for the work W that must be done by an external agent on the second particle in order to increase the radius of the circle of motion, centred at P, to* r_2.

Solution:

The centripetal force required for the particle of mass m to move in a circular orbit of radius r with speed v is $\frac{mv^2}{r}$. This is provided by the Coulomb force of attraction between the charges $+Q$ and $-q$. It is

$$F_c = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}.$$

We have the equation

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2},$$

or

$$mv^2 = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r}.$$

The energy of the particle moving in a circular orbit in the attractive Coulomb field will be

$$E = \frac{mv^2}{2} - \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r} = \frac{1}{2}\times\frac{1}{4\pi\varepsilon_0}\frac{Qq}{r} - \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r}$$

$$= -\frac{1}{2}\times\frac{1}{4\pi\varepsilon_0}\frac{Qq}{r}.$$

Therefore, the work done by an external agent required for increasing the radius of the circle of motion from r_1 to r_2 will be

$$W = E\left(r_2\right) - E\left(r_1\right) = \frac{1}{2}\times\frac{Qq}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$= \frac{1}{2}\times\frac{Qq}{4\pi\varepsilon_0}\left(\frac{r_2 - r_1}{r_2 r_1}\right).$$