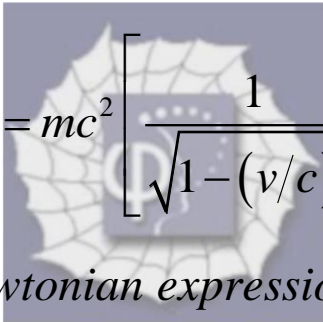


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**Problem 30.9 (RHK)**

(a) We have to find, applying Newtonian mechanics, the potential difference across which an electron must fall so that it may acquire a speed  $v$  equal to the speed of light  $c$ . (b) The Newtonian mechanics fails as  $v \rightarrow c$ , therefore applying the correct relativistic expression for the kinetic energy


$$K = mc^2 \left[ \frac{1}{\sqrt{1 - (v/c)^2}} - 1 \right]$$

in place of the Newtonian expression  $K = \frac{1}{2}mv^2$ , we have to find the actual electron speed acquired in falling through the potential difference computed in (a). We have to express this speed as an appropriate fraction of the speed of light.

**Solution:**

Across a potential difference  $V$  the potential energy of an electron will be

$$E = eV.$$

So when the electron falls through the potential difference  $V$  it will acquire kinetic energy equal to  $E$ . If the speed of the electron on falling through potential  $V$  is  $v$ , we have

$$\frac{1}{2}mv^2 = eV,$$

where  $m$  is the mass of electron. Therefore, for  $v$  to be equal to  $c$ , the potential difference  $V$  will have to be

$$V = \frac{mc^2}{2e} = \frac{9.1 \times 10^{-31} \times (3 \times 10^8)^2}{2 \times 1.6 \times 10^{-19}} \text{ V} \\ = 2.56 \times 10^5 \text{ V.}$$

(b)

We will next use the relativistic expression for kinetic energy

$$K = mc^2 \left[ \frac{1}{\sqrt{1 - (v/c)^2}} - 1 \right].$$

The actual electron speed acquired in falling through the potential difference

$$V = \frac{mc^2}{2e},$$

will therefore be given by the equation

$$\frac{mc^2}{2} = mc^2 \left[ \frac{1}{\sqrt{1-(v/c)^2}} - 1 \right].$$

Or

$$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{3}{2},$$

or

$$1 - \frac{v^2}{c^2} = \frac{4}{9}.$$

And

$$v = \sqrt{\frac{5}{9}} c = 0.745c = 2.24 \times 10^8 \text{ m s}^{-1}.$$

