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## Problem 30.9 (RHK)

(a) *We have to find, applying Newtonian mechanics, the potential difference across which an electron must fall so that it may acquire a speed v equal to the speed of light c*. (b) T*he Newtonian mechanics fails as* $v \to c$, *therefore applying the correct relativistic expression for the kinetic energy*

$$K = mc^2 \left[ \frac{1}{\sqrt{1-(v/c)^2}} - 1 \right]$$

*in place of the Newtonian expression* $K = \frac{1}{2}mv^2$, *we have to find the actual electron speed acquired in falling through the potential difference computed in* (a). *We have to express this speed as an appropriate fraction of the speed of light.*

## Solution:

Across a potential difference $V$ the potential energy of an electron will be

$E = eV.$

So when the electron falls through the potential difference *V* it will acquire kinetic energy equal to *E*. If the speed of the electron on falling through potential *V* is *v*, we have

$$\frac{1}{2}mv^2 = eV,$$

where *m* is the mass of electron. Therefore, for *v* to be equal to *c*, the potential difference *V* will have to be

$$V = \frac{mc^2}{2e} = \frac{9.1\times10^{-31}\times\left(3\times10^8\right)^2}{2\times1.6\times10^{-19}}\text{ V}$$
$$= 2.56\times10^5\text{ V.}$$

(b)

We will next use the relativistic expression for kinetic energy

$$K = mc^2\left[\frac{1}{\sqrt{1-\left(v/c\right)^2}} - 1\right].$$

The actual electron speed acquired in falling through the potential difference

$$V = \frac{mc^2}{2e},$$

will therefore be given by the equation

$$\frac{mc^2}{2} = mc^2\left[\frac{1}{\sqrt{1-(v/c)^2}} - 1\right].$$

Or

$$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{3}{2},$$

or

$$1 - \frac{v^2}{c^2} = \frac{4}{9}\ .$$

And

$$v = \sqrt{\frac{5}{9}}\,c = 0.745c = 2.24\times10^8\ \text{m s}^{-1}.$$