333.

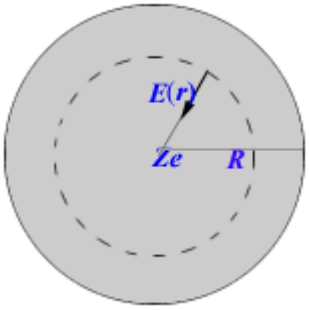
Problem 29.49 (RHK)

In a 1911 paper, Ernst Rutherford said:

"*In order to form some idea of the forces required to deflect an alpha particle through a large angle, consider an atom containing a point positive charge Ze at its centre and surrounded by a distribution of negative electricity, –Ze uniformly distributed within a sphere of radius R. The electric field E … at a distance r from the centre for a point inside the atom is*

$$E = \frac{Ze}{4\pi\varepsilon_0}\left(\frac{1}{r^2} - \frac{r}{R^3}\right).$$"

Solution:



As charge $-Ze$ is uniformly distributed over a sphere of radius R, the volume charge density inside the sphere is

$$\rho = -\frac{3Ze}{4\pi R^3}.$$

Point charge of amount $+Ze$ is located at the centre of the sphere. Therefore, the charge contained inside a sphere of radius *r* is

$$q(r) = Ze - Ze\left(\frac{r^3}{R^3}\right) = Ze\left(1 - \frac{r^3}{R^3}\right).$$

Therefore, electric field at a distance r from the centre of the sphere will be

$$E(r) = \frac{q(r)}{4\pi\varepsilon_0 r^2} = \frac{Ze}{4\pi\varepsilon_0}\left(\frac{1}{r^2} - \frac{r}{R^3}\right).$$