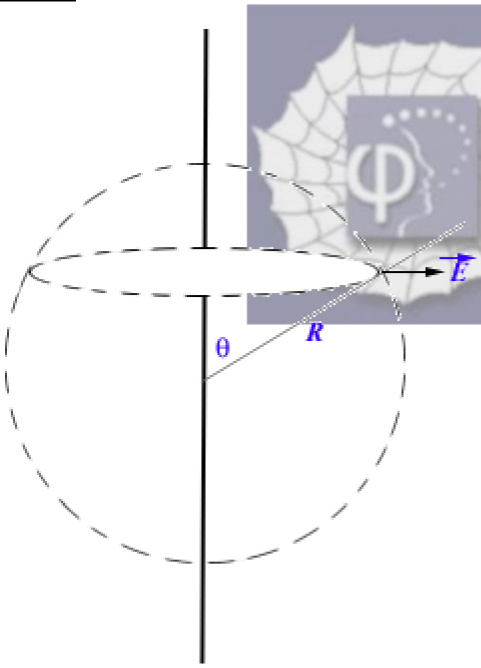
332.

Problem 29.48 (RHK)

By constructing a spherical Gaussian surface centred on an infinite line of charge and by calculating the flux through the sphere we will show that the Gauss' law is satisfied.

Solution:



Electric field at a distance r from an infinite line of charge of linear charge density λ is

$$E(r) = \frac{\lambda}{2\pi\varepsilon_0 r},$$

and its direction is normal to the line.

We consider a spherical Gaussian surface of radius R centred on the line as shown in the figure. Flux through Gaussian surface of area

$2\pi R \sin\theta R d\theta$

will be

$$\Delta\Phi = 2\pi R\sin\theta R d\theta \frac{\vec{R}.\vec{E}}{R} = 2\pi R\sin\theta R d\theta \frac{\lambda\sin\theta}{2\pi\varepsilon_0 R\sin\theta}$$

$$= \frac{\lambda\sin\theta R d\theta}{\varepsilon_0}.$$

Therefore,

$$\Phi(R) = \int_0^{\pi} \frac{\lambda R\sin\theta d\theta}{\varepsilon_0} = \frac{2\lambda R}{\varepsilon_0}.$$

And charge contained inside the sphere is $2R\lambda$.

Therefore, we have

$\varepsilon_0\Phi(R) = q$,

and Gauss' law is verified.