330.

Problem 29.47 (RHK)

A solid nonconducting sphere of radius R carries a non-uniform charge distribution, the charge density being $\rho = \rho_s r/R$, where ρ_s is a constant and r is the distance from the sphere. We have to show that (a) the total charge on the sphere is $Q = \pi\rho_s R^3$ and (b) the electric field inside the sphere is given by

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^4}r^2.$$

Solution:

(a)

The charge density in the solid nonconducting sphere of radius R is given by the function

$\rho = \rho_s r/R,$

where ρ_s is a constant and r is the distance from the sphere.

The total charge in the sphere can be obtained by integrating the density over the volume of the sphere

$$Q = \int_0^R 4\pi r^2 \rho(r) dr = 4\pi\rho_s \int_0^R \frac{r^3}{R} dr = \pi\rho_s R^3.$$

(b)

The electric field at a point which is at a distance r from the centre of the sphere will be determined by the total charge contained inside the sphere of radius *r*. We therefore have

$$E(r) = \frac{1}{4\pi\varepsilon_0 r^2} \int_0^r 4\pi r'^2 \rho_s \frac{r'}{R} dr' = \frac{\rho_s r^2}{4\varepsilon_0 R} = \frac{Q r^2}{4\pi\varepsilon_0 R^4}.$$