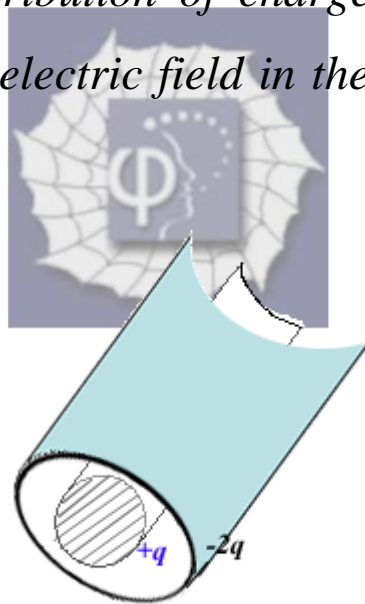


328.

Problem 29.29 (RHK)

A very long conducting cylinder (length L) carrying a total charge $+q$ is surrounded by a conducting cylindrical shell with total charge $-2q$, as shown in cross-section in the figure. We have to use Gauss' law to find (a) the electric field at points outside the conducting shell, (b) the distribution of charge on the conducting shell, and (c) the electric field in the region between the cylinders.



Solution:

We will assume that the lengths of the conducting cylinder and the conducting cylindrical shell are very large compared to their cross-sectional dimensions, and we can use infinite length approximation for determining the electrical field by neglecting the effect of the ends.

In this approximation the electrical field within the space enclosed by the cylindrical shell and the cylindrical conductor and that outside the shell will be normal to the cylindrical axis.

(a)

We consider a cylindrical Gaussian surface of radius r and length L enclosing the shell. Let the electric field be $E(r)$. The normal flux through the surface will be

$$\Phi(r) = 2\pi rLE(r).$$

The amount of charge enclosed by this surface will be

$$Q = +q - 2q = -q.$$

By Gauss' law we have

$$\Phi = \frac{Q}{\epsilon_0}.$$

Therefore,

$$E(r) = -\frac{q}{2\pi\epsilon_0 L}.$$

The minus indicates that the field points inward that is toward the axis of the cylindrical conductor.

(b)



As there cannot be any charge within the cylindrical conductor, charge $+q$ will be uniformly distributed on its outer surface.

As there cannot be electric field within the conducting cylindrical shell, the total charge $-2q$ on the conducting cylindrical shell will get distributed such that charge on its inner surface is $-q$ and charge on its outer surface is also $-q$.

(c)

For finding the electric field in the region between the cylinders we consider a Gaussian surface that enclosed the cylindrical conductor but is enclosed by the conducting cylindrical shell.

As the total charge enclosed by this surface is $+q$, the electrical field within the region between the cylinders will be

$$E(r) = \frac{q}{2\pi r L \epsilon_0} .$$

Direction of the electric field will be the outward normal to the axis of the cylinders.