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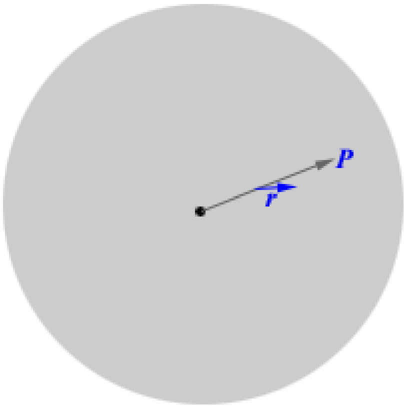
Problem 24.57P (HRW)

A nonconducting sphere has a uniform volume charge density ρ. *Let* $\vec{r}$ *be the vector from the centre of the sphere to a general point P within the sphere. We have to show* (a) *that the electric field at P is given by* $\vec{E} = \rho\vec{r}/3\varepsilon_0$. (*Note that the result is independent of the radius of the sphere.*) (b) *A spherical cavity is hollowed out of the sphere as shown in the figure. Using the superposition concepts, we have to show that the electric field at all points within the cavity is* $\vec{E} = \rho\vec{a}/3\varepsilon_0$ (*uniform field*), *where* $\vec{a}$ *is the position vector pointing from the centre of the sphere to the centre of the cavity.* (*Note that this result is independent of the radius of the sphere and also the radius of the cavity*.)

Solution:

A nonconducting sphere has a uniform volume charge density ρ. Let $\vec{r}$ be the vector from the centre of the sphere to a general point *P* within the sphere.



We will calculate the electric field at *P*. The quantity of charge contained inside a sphere of radius *r* centred at the centre of the sphere will be

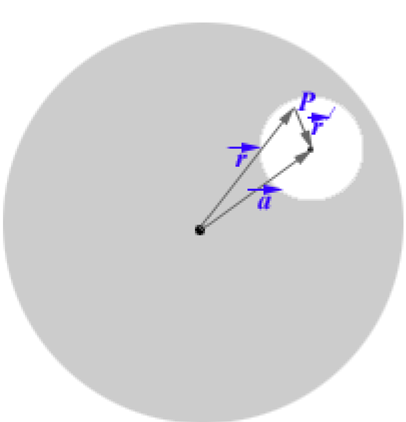
$$Q(r) = \frac{4\pi r^3 \rho}{3} \ .$$

Therefore, the electric field at *P* will be

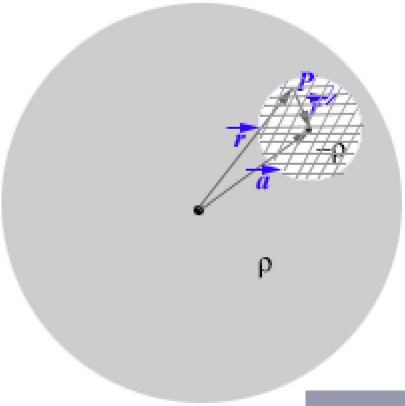
$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \times \frac{Q(r)}{r^2} \times \frac{\vec{r}}{r} = \frac{1}{4\pi\varepsilon_0} \times \frac{4\pi r^3 \rho}{3r^2} \times \frac{\vec{r}}{r} = \frac{\rho \vec{r}}{3\varepsilon_0}.$$

Note that it is independent of the size of the uniformly charged sphere.

We next consider a spherical cavity inside the sphere with centre at a distance $\vec{a}$ from the centre of the sphere. We will like to find the electric field at a point *P* within the cavity.



By superposition, we can consider the charged sphere with cavity as the sphere with charge density ρ with an imbedded sphere of the size of the cavity with charge density $-\rho$.



By superposition principle the electric field at point P within the cavity will be

$$\vec{E}(P) = \frac{\rho \vec{r}}{3\varepsilon_0} + \frac{\rho \vec{r}'}{3\varepsilon_0} = \frac{\rho \vec{a}}{3\varepsilon_0}.$$