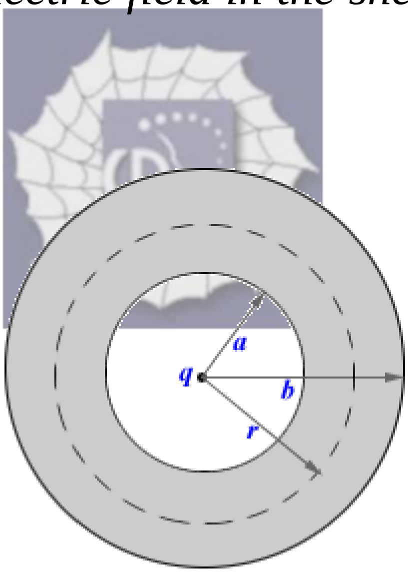
325.

Problem 24.55P (HRW)

A nonconducting spherical shell, of inner radius a and outer radius b, has a volume charge density $\rho = A/r$ (*within its thickness*), *where A is a constant and r is the distance from the centre of the shell. In addition, a point charge q is located at the centre. We have to find the value of A if the electric field in the shell* $(a \le r \le b)$ *is to be uniform.*



Solution:

The inner and outer radii of the nonconducting spherical shell are *a* and *b*, respectively. It is given that the shell contains a spherically symmetric charge distribution. The volume charge density within the shell is described by the function

$$\rho = \frac{A}{r},$$

where *A* is a constant. A point charge q is placed at the centre of the spherical shell.

We have to determine the constant *A* for which the electric field inside the shell, $a \le r \le b$, will be uniform, that is it will not depend on *r*.

We will apply Gauss' law for calculating the electric field at a distance *r* from the centre of the shell and within the shell. Let us consider a spherical surface of radius *r* such that $a \le r \le b$. Because of the spherical symmetry the field can be a function of *r* only. The outward flux on this surface will be

$$\Phi(r) = 4\pi r^2 E(r).$$

The total amount of charge contained within this surface will be

$$Q = \int_a^r 4\pi r^2 \rho(r)\,dr + q$$

$$= \int_a^r 4\pi r^2 \times \frac{A}{r} dr + q = 2\pi A\left(r^2 - a^2\right) + q.$$

Gauss' law sates that

$$\Phi(r) = \frac{Q}{\varepsilon_0}.$$

Therefore,

$$E(r) = \frac{A}{2\varepsilon_0} + \left(\frac{q}{4\pi\varepsilon_0} - \frac{a^2 A}{2\varepsilon_0} \right) \times \frac{1}{r^2}.$$

For the field $E(r)$ to be uniform, the condition is

$$\left(\frac{q}{4\pi\varepsilon_0} - \frac{a^2 A}{2\varepsilon_0} \right) = 0,$$

or

$$A = \frac{q}{2\pi a^2}.$$