Problem 29.11 (RHK)

"Gauss' law for gravitation" is

$$\frac{1}{4\pi G}\Phi_g = \frac{1}{4\pi G}\mathbf{\tilde{N}}^{\mathbf{r}}.d\mathbf{\tilde{A}} = -m,$$

where m is the enclosed mass and G is the universal gravitational constant. We will derive Newton's law of gravitation from this and explain the significance of the

minus sign.



Solution:

Consider a Gaussian sphere, a sphere of radius r centred at the mass point of mass, m. We assume that the gravitational field $\frac{1}{2}$ due to the mass point is radial and its magnitude is equal for all points at a distance r from it but the direction of the gravitational field varies and is toward the mass point.

We calculate the gravitational flux under the above assumptions. It will be

$$\Phi_g = -4\pi r^2 g \; .$$

The Gauss' law for gravitation is

315.

$$\frac{1}{4\pi G}\Phi_g = \frac{1}{4\pi G}\int g dA = -m.$$

We therefore have

$$-\frac{r^2g}{G}=-m,$$

And

$$g=\frac{Gm}{r^2},$$

which is the Newton's law of gravitation. The significance of minus sign in the Gauss' law for gravitation is that the gravitational field due to a mass point is radial and is attractive.

