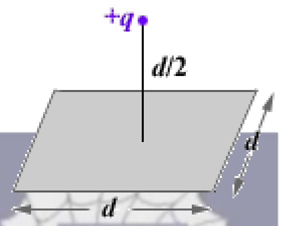
313.

Problem 29.7 (RHK)

A point charge $+q$ *is a distance* $d/2$ *from a square surface of side d and is directly above the centre of the square as shown in the figure. We have to find the electric flux through the square.*



Solution:

We will calculate the flux through the square surface in two ways.

(a)

We will first consider the square surface as one of the six faces of a cube of edge *d*. Flux through a Gaussian surface of a cube containing charge $+q$ will be

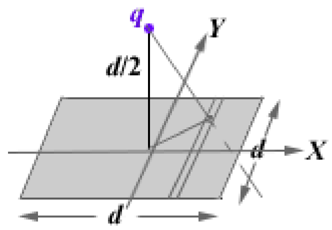
$$\Phi'_E = \frac{q}{\varepsilon_0},$$

and therefore the flux through one of the faces, as cube has six faces symmetric with respect to its centre, will be

$$\Phi_E = \frac{\Phi'_E}{6} = \frac{q}{6\varepsilon_0}.$$

(b)

Our second answer is a direct calculation of electric field flux at the surface of the square. We will first calculate the electric field vector at point (*x*, *y*) on the plane.



Let us consider an infinitesimal element of surface of area $dxdy$ at the point (*x*, *y*). The magnitude of the electric field at that point due to a point charge $+q$ will be

$$E(x,y) = \frac{q}{4\pi\varepsilon_0\left(x^2+y^2+d^2/4\right)},$$

And the cosine of the angle between the outward normal to the surface and the direction of the electric field at the point (*x*, *y*) is

$$\frac{d}{2\left(x^2+y^2+d^2/4\right)^{1/2}}.$$

Therefore,

$$\Delta\Phi_E(x,y) = \frac{q}{4\pi\varepsilon_0}\times\frac{d}{2}\frac{dxdy}{\left(x^2+y^2+d^2/4\right)^{3/2}}.$$

And,

$$\Phi_E = \frac{q}{4\pi\varepsilon_0} \times \frac{d}{2} \times \int_{-d/2}^{d/2} dx \int_{-d/2}^{d/2} dy \frac{1}{\left(x^2 + y^2 + d^2/4\right)^{3/2}}$$

$$= \frac{q}{4\pi\varepsilon_0} \times \frac{d}{2} \times 2 \times 2 \times \int_{0}^{d/2} dx \int_{0}^{d/2} dy \frac{1}{\left(x^2 + y^2 + d^2/4\right)^{3/2}}.$$

On performing integration by appropriate substitution, we find

$$\int_0^{d/2} \frac{dy}{\left(x^2 + y^2 + d^2/4\right)^{3/2}} = \frac{1}{\left(x^2 + d^2/4\right)} \times \frac{d/2}{\left(x^2 + d^2/2\right)^{1/2}}.$$

And

$$\Phi_E = \frac{qd^2}{4\pi\varepsilon_0} \int_0^{d/2} \frac{dx}{\left(x^2 + d^2/4\right)\left(x^2 + d^2/2\right)^{1/2}}.$$

On performing integration by appropriate substitution, we find

$$\int_0^{d/2} \frac{dx}{\left(x^2 + d^2/4\right)\left(x^2 + d^2/2\right)^{1/2}} = \frac{4}{d^2} \times \frac{\pi}{6}.$$

And

$$\Phi_E = \frac{q}{6\varepsilon_0}.$$