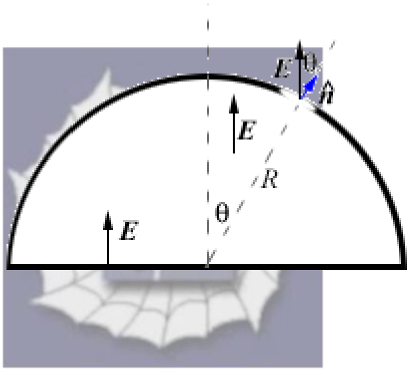
311.

Problem 29.3 (RHK)

We will calculate Φ_E *through* (a) *the flat base and* (b) *the curved surface of a hemisphere of radius R. The field* $\vec{E}$ *is uniform and parallel to the axis of the hemisphere, and the lines of force enter through the flat base.*



Solution:

(a)

As the area of the flat base of the hemisphere of radius R is πR^2 and the direction of the outward normal to the surface is opposite to that of the electric field $\vec{E}$, the flux entering through the base will be

$$\Phi_E^{(a)} = -\pi R^2 E.$$

(b)

The flux leaving through the top surface of the hemisphere will be $\pi R^2 E$ as the amount of flux that enters the base of the hemisphere will leave it from its enclosing top surface. We will calculate this result by explicitly using the definition of flux.

Let us set up a spherical polar coordinate system as shown in the figure. Consider an infinitesimal element of area on the surface at angular coordinates θ, φ, $\Delta a = R^2 \sin\theta d\theta d\varphi$. The normal vector to the surface element will be in the radial direction. The surface element vector is

$\Delta \vec{a} = R^2 \sin\theta d\theta d\varphi \hat{r}$.

Therefore,

$\vec{E}.\Delta\vec{a} = ER^2 \cos\theta \sin\theta d\theta d\varphi.$

The total flux leaving the surface of the hemisphere will be

$$\Phi_E^{(b)} = \int_0^{\pi/2} d\theta \int_0^{2\pi} ER^2 \sin\theta \cos\theta d\varphi$$

$$= 2\pi ER^2 \int_0^{\pi/2} \sin\theta \cos\theta d\theta = \pi ER^2.$$