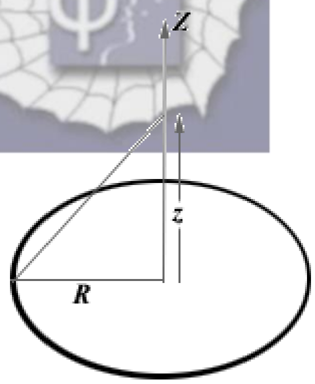
306.

Problem 28.46 (RHK)

An electron is constrained to move along the axis of uniformly charged ring of radius with total charge q. We have to show that the electron can perform small oscillations, through the centre of the ring, with a frequency given by

$$\omega = \sqrt{\frac{eq}{4\pi\varepsilon_0 mR^3}} \; .$$



Solution:

We note that electric field at a distance z from the centre of a ring of radius R and total charge q is

$$\vec{E} = \frac{q}{4\pi\varepsilon_0} \times \frac{z}{\left(z^2 + R^2\right)^{3/2}}\,\hat{z}$$

We consider motion of an electron in the direction of the z-axis and close to the plane of the ring. That is approximate

$$\frac{z}{R} = 1 .$$

As electrons are negatively charged and the charge of an electron is

$$-e,\ e = 1.6\times 10^{-19}\ \mathrm{C} .$$

Force on an electron moving along the axis of the ring will be

$$\vec{F} = -e\vec{E} = -\frac{qez}{4\pi\varepsilon_0\left(z^2+R^2\right)^{3/2}}\hat{z} .$$

Close to the plane of the ring we can approximate $\vec{F}$ by

$$\vec{F}\ ;\ -\frac{qez}{4\pi\varepsilon_0 R^3}\hat{z} .$$

Equation of motion of an electron of mass m moving along the axis of the ring close to its plane will be

$$m\frac{d^2z}{dt^2} = -\frac{qez}{4\pi\varepsilon_0 R^3}.$$

Or

$$\frac{d^2z}{dt^2} + \frac{qe}{4\pi\varepsilon_0 m R^3}z = 0.$$

It is an equation of simple harmonic motion (SHM) with period

$$\omega = \sqrt{\frac{eq}{4\pi\varepsilon_0 m R^3}} \ .$$