302.

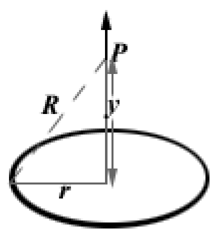
Problem 28.35 (RHK)

A nonconducting hemispherical cup of inner radius R has a total charge q spread uniformly over its inner surface. We have to find the electric field at the centre of curvature.

Solution:

We are given a nonconducting hemispherical cup of inner radius *R* that has a total charge *q* spread uniformly over its inner surface. We will calculate the electric field at the centre of curvature of the hemisphere, which is at the point *P*, in two steps.

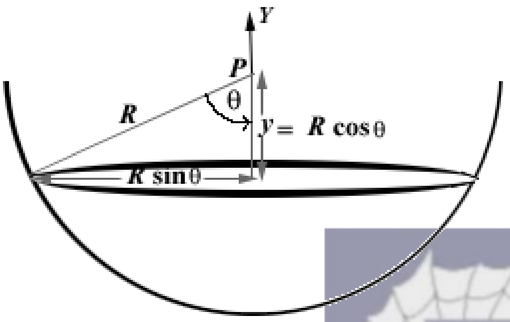
In the first step we will find the electric field at a point y unit above the centre of a ring of radius r and on the axis passing through its centre.



Let the charge on the ring be ΔQ. From the symmetry of the charge distribution we note that the electric field at P will be along the y-axis and its magnitude will be

$$\Delta E(y) = \frac{\Delta Q}{4\pi\varepsilon_0 R^2} \times \frac{y}{R} = \frac{\Delta Q\, y}{4\pi\varepsilon_0 R^3} \; .$$

In the second step we will consider the hemispherical cup as a stack of rings, as shown in the figure. Let us consider a ring at angle θ from the azimuth, as shown in the figure.



Perpendicular distance of the ring from the centre of curvature of the hemispherical cup is $R\cos\theta$. Area of this ring on the surface of the hemispherical cup will be

$\Delta A = 2\pi R \sin\theta \times R d\theta = 2\pi R^2 \sin\theta d\theta.$

As the amount of charge on the inner surface of the hemispherical cup is q, the surface charge density is

$$\sigma = \frac{q}{2\pi R^2} \; .$$

Charge on the ring, $\Delta Q = \Delta A \sigma$.

Using the result of step 1, we have

$$\Delta E(y) = \frac{\Delta Q\, y}{4\pi\varepsilon_0 R^3} = \frac{2\pi R^2 \sin\theta d\theta \sigma R \cos\theta}{4\pi\varepsilon_0 R^3}$$

$$= \frac{\sigma \sin 2\theta d\theta}{4\varepsilon_0} \; .$$

By integrating $\Delta E(y)$ from 0 to $\pi/2$, we calculate the electric field at the centre of curvature of the hemispherical cup. We have

$$E=\int_0^{\pi/2}\frac{\sigma\sin 2\theta d\theta}{4\varepsilon_0}=\frac{\sigma}{4\varepsilon_0}\ .$$

Substituting for σ, we find

$E=\dfrac{q}{8\pi\varepsilon_0 R^2}$. Its direction is along the perpendicular axis passing through the centre of curvature of the hemisphere.