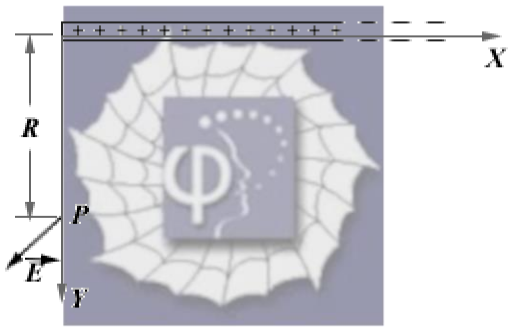
301.

Problem 28.34 (RHK)

A "semi-infinite" insulating rod carries a constant charge per unit length of λ*. We have to show that the electric field at the point P makes an angle of* 45^0 *with the rod and that this result is independent of the distance R.*



Solution:

We will calculate the *x*-component and the *y*-component of the $\vec{E}$ field at the point *P*, as shown in the figure, due to the "semi-infinite" insulating rod carrying uniform linear charge of density λ.

Consider an infinitesimal charge element of length dx at distance *x* from the end of the rod as shown in the figure. As the direction of electric field due to a charge element is along the line joining the element to the point where field is to be calculated, the *x*-component of the field will be

$$dE_X = \frac{\lambda dx}{4\pi\varepsilon_0\left(x^2+R^2\right)} \times \left(-\frac{x}{\left(x^2+R^2\right)^{1/2}} \right)$$

$$= -\frac{\lambda x dx}{4\pi\varepsilon_0\left(x^2+R^2\right)^{3/2}} \ .$$

Integrating dE_X over the length of the "semi-infinite" rod that is with respect to the variable x from 0 to ∞, we get

$$E_X = -\frac{\lambda}{4\pi\varepsilon_0}\int_0^\infty \frac{xdx}{\left(x^2+R^2\right)^{3/2}} = -\frac{\lambda}{4\pi\varepsilon_0}\times\frac{1}{2}\left.\frac{\left(x^2+R^2\right)^{-1/2}}{(-1/2)}\right|_0^\infty$$

$$= -\frac{\lambda}{4\pi\varepsilon_0}\times\frac{1}{R} = -\frac{\lambda}{4\pi\varepsilon_0 R} \ .$$

Similarly,

$$dE_Y = \frac{\lambda dx}{4\pi\varepsilon_0\left(x^2+R^2\right)} \times \frac{R}{\left(x^2+R^2\right)^{1/2}} \ .$$

And

$$E_Y = \int_0^\infty \frac{\lambda dx}{4\pi\varepsilon_0\left(x^2+R^2\right)} \times \frac{R}{\left(x^2+R^2\right)^{1/2}}$$

$$= \frac{\lambda R}{4\pi\varepsilon_0}\int_0^\infty \frac{dx}{\left(x^2+R^2\right)^{3/2}} \ .$$

For calculating this integral, we make the substitution

$x = R\tan\theta,$

$dx = R\sec^2\theta d\theta$.

We have

$$E_Y = \frac{\lambda R}{4\pi\varepsilon_0}\int_0^{\pi/2}\frac{R\sec^2\theta d\theta}{R^3\sec^3\theta} = \frac{\lambda}{4\pi\varepsilon_0 R}\int_0^{\pi/2}\cos\theta d\theta = \frac{\lambda}{4\pi\varepsilon_0 R} .$$

We note that

$|E_X| = |E_Y|$ for all *R*.

Therefore, the angle that the electric field vector makes at *P* is

$$\tan\theta = \frac{|E_Y|}{|E_X|} = 1 .$$

It implies that $\theta = 45^0$ and is independent of *R* that is the distance of the point *P* from the edge of the “semi-infinite” rod.