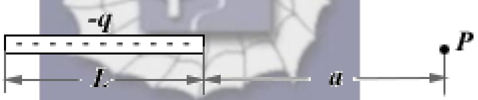
300.

<u>Problem 28.32 (RHK)</u>

An insulating rod of length L has charge $-q$ *uniformly distributed along its length, as shown in the figure.* (a) *We have to find the linear charge density of the rod;* (b) *the electric field at point P a distance a from the end of the rod.* (c) *If P were very far from the rod compared to L, the rod would look like a point charge. We have to show that our answer for* (b) *reduces to the electric field of a point charge for* a ? L.



<u>Solution:</u>

(a)

As charge $-q$ is distributed uniformly over length L, the linear charge density will be

$$\lambda = -\frac{q}{L}.$$

(b)

Electric field at *P* due to an infinitesimal length dx of the rod at a distance *x* from its far end will be

$$dE = \frac{\lambda dx}{4\pi\varepsilon_0 \left(L - x + a\right)^2}.$$

And the electric field at P due to the charged rod will be given by the integral

$$E = \int_0^L \frac{\lambda dx}{4\pi\varepsilon_0 \left(L - x + a\right)^2} = \frac{\lambda}{4\pi\varepsilon_0}\int_0^L dx\left(L - x + a\right)^{-2}$$

$$= \frac{\lambda}{4\pi\varepsilon_0}(-)\frac{1}{\left(x - L - a\right)}\Bigg|_0^L$$

$$= -\frac{\lambda}{4\pi\varepsilon_0}\left(\frac{-1}{a} + \frac{1}{L + a}\right)$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\frac{L}{a\left(L + a\right)} = -\frac{q}{4\pi\varepsilon_0 a\left(L + a\right)}.$$

As the rod is negatively charged, the direction of the electric field will be towards it.

(c)

For a ? L, we note that the electric field can be approximated by the expression

$$E \; ; \; -\frac{q}{4\pi\varepsilon_0 a^2}\;.$$

It is the field due to a point charge.