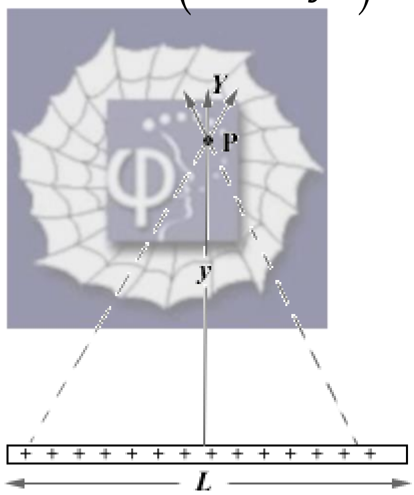
299.

Problem 28.31 (RHK)

A thin non-conducting rod of finite length L carries a total charge q, spread uniformly along it. We have to show that E at point P on the perpendicular bisector is given by

$$E = \frac{q}{2\pi\varepsilon_0 y}\frac{1}{\left(L^2+4y^2\right)^{1/2}} \ .$$



Solution:

As charge q is uniformly distributed over a length L, the linear charge density is

$$\lambda = \frac{q}{L} \ .$$

Looking at the direction of the electric field vectors at point P due to the symmetrically placed charge elements on the two sides of the rod, we note that the resultant electric field will be along the Y-axis.

The magnitude of the electric field at point P will be given by the integral

$$E(y) = 2\int_0^{L/2} \frac{\lambda dx}{4\pi\varepsilon_0\left(x^2+y^2\right)} \frac{y}{\left(x^2+y^2\right)^{1/2}}$$

$$= \frac{2qy}{4\pi\varepsilon_0 L}\int_0^{L/2} \frac{dx}{\left(x^2+y^2\right)^{3/2}} \ .$$

For calculating this integral, we make the substitution

$x = y\tan\theta.$

We therefore have

$dx = y\sec^2\theta d\theta$.

And

$$E(y) = \frac{2qy}{4\pi\varepsilon_0 L}\int_0^{\tan^{-1}(L/2y)} \frac{y\sec^2\theta d\theta}{y^3\sec^3\theta}$$

$$= \frac{q}{2\pi\varepsilon_0 Ly}\int_0^{\tan^{-1}(L/2y)} \cos\theta d\theta = \frac{q}{2\pi\varepsilon_0 Ly}\sin\left(\tan^{-1}\left(L/2y\right)\right)$$

$$= \frac{q}{2\pi\varepsilon_0 Ly}\times\frac{L}{\left(L^2+4y^2\right)^{1/2}}.$$

Or

$$E\left(y\right)=\frac{q}{2\pi\varepsilon_0 y}\frac{1}{\left(L^2+4y^2\right)^{1/2}}\ .$$