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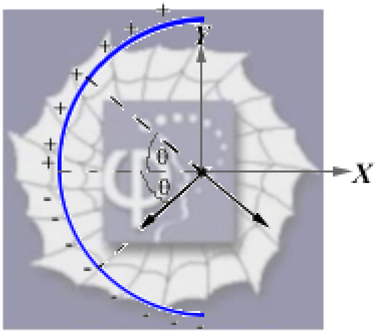
Problem 28.30 (RHK)

A thin glass rod is bent into a semicircle of radius r. A charge $+q$ is uniformly distributed along the upper half and a charge $-q$ is uniformly distributed along the lower half, as shown in the figure. We have to find the electric field $\vec{E}$ at P, the centre of the semicircle.



Solution:

As charge $+q$ is uniformly distributed along the upper half of the circumference of a uniform glass rod of radius R, the linear charge density will be

$$\lambda_+ = \frac{q}{(\pi R/2)} = \frac{2q}{\pi R}.$$

Similarly, as charge $-q$ is uniformly distributed along the lower half of the circumference of a uniform glass rod of radius *R*, the linear charge density will be

$$\lambda_{-} = \frac{-q}{(\pi R/2)} = \frac{-2q}{\pi R}.$$

From the diagram we note that the electric field along the *X*-axis due to the upper half charges and the lower half charges at the centre of the circle will cancel each other. Therefore, the net electric field at the centre of the circle will be along the *Y*-axis. From the diagram we note that the net electric field at the centre of the circle will be along the –*Y*-axis. Its magnitude will be

$$E = 2\int_0^{\pi/2} \frac{\lambda_{+} R d\theta}{4\pi\varepsilon_0 R^2} \sin\theta = \frac{2\lambda_{+}}{4\pi\varepsilon_0 R} = \frac{2(2q/\pi R)}{4\pi\varepsilon_0 R}$$

$$= \frac{q}{\pi^2 \varepsilon_0 R^2}.$$