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Problem 28.27 (RHK)

We have to find (a) *the total charge q that a disk of radius* 2.50 cm *must carry in order that the electric field on the surface of the disk at its centre equals the value at which air breaks down electrically, producing sparks.* (b) *Suppose that each atom at the surface has an effective cross-sectional area of* $0.015\ \text{nm}^2$. *We have to estimate the number of atoms at the disk's surface.* (c) *If the charge in* (a) *results from one excess electron carried by some of the surface atoms, we have to find the fraction of the surface atoms that must be so charged.*

Solution:

(a)

Electric breakdown in air takes place if the electric field strength exceeds $3\times10^6\ \text{N C}^{-1}$.

Let us consider a uniformly charged disk of radius R and total charge q. The surface charge density will be

$$\sigma = \frac{q}{\pi R^2}\ .$$

The electric field strength at the centre of a uniformly charged disk is equal to the electric field of an infinite plate having uniform charge density σ. That is

$$E = \frac{\sigma}{2\varepsilon_0} = \frac{q}{2\pi\varepsilon_0 R^2} \ .$$

The charge q on a disk of radius R that will produce breakdown of air will be

$$q = \frac{4\pi\varepsilon_0}{2} R^2 E_{spark} \, .$$

It is given that the radius of the disk is 2.50 cm. Therefore, the total charge q that it must have for air to breakdown at its centre will be

$$q = \frac{1}{2 \times 8.99 \times 10^9} \times \left(2.5 \times 10^{-2}\right)^2 \times 3 \times 10^6 \text{ C}$$
$$= 104 \text{ nC}.$$

(b)

It is given that the effective cross-sectional area of each atom on the surface of the disk is 0.015 nm^2. Therefore, the total number of atoms on the disk is

$$N = \frac{\pi\left(2.5 \times 10^{-2}\right)^2}{15 \times 10^{-21}} = 1.31 \times 10^{17}.$$

(c)

It is given that the charge on the disk is due to those atoms which carry one excess electron, that is have a charge $e = 1.6 \times 10^{-19}$ C. Therefore, the total number of charged atoms on the disk is

$$N_c = \frac{1.043 \times 10^{-7}\ \text{C}}{1.6 \times 10^{-19}\ \text{C}} = 0.652 \times 10^{12}.$$

Fraction of the surface atoms that are charged will be

$$f = \frac{N_c}{N} = \frac{0.652 \times 10^{12}}{1.31 \times 10^{17}} = 4.97 \times 10^{-6}.$$