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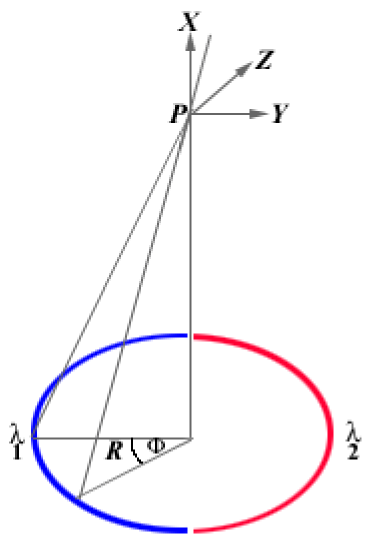
Problem 28.15 (RHK)

Consider a ring of charge. Suppose that the charge q *is not distributed uniformly over the ring but that charge* q_1 *is distributed uniformly over half the circumference and charge* q_2 *is distributed* uniformly *over the other half. Let* $q_1 + q_2 = q$. *We have to find* (a) *the component of the electric field at any point on the axis directed along the axis and compare it with the uniform case;* (b) *the component of the electric field at any point on the axis perpendicular to the axis and compare it with the uniform case.*



Solution:

As shown in the figure, on the circumference of a ring of radius *R* divided into two equal parts charge q_1 is distributed uniformly on part and charge q_2 is distributed uniformly on the other part. We will calculate the electric field at point P on the axis of the ring. A coordinate system has been fixed as shown in the figure.

Linear charge densities on the two halves of the ring will be

$$\lambda_1 = \frac{q_1}{\pi R}, \text{ and } \lambda_2 = \frac{q_2}{\pi R} .$$

The component of the electric field at point *P* on the axis directed along it that is along the *X*-direction will be

$$E_x = \int_0^{\pi R} \frac{\lambda_1 ds}{4\pi\varepsilon_0\left(x^2+R^2\right)} \times \frac{x}{\left(x^2+R^2\right)^{1/2}}$$

$$+ \int_{\pi R}^{2\pi R} \frac{\lambda_2 ds}{4\pi\varepsilon_0\left(x^2+R^2\right)} \times \frac{x}{\left(x^2+R^2\right)^{1/2}}$$

$$= \left(\lambda_1 \pi R + \lambda_2 \pi R\right)\frac{1}{4\pi\varepsilon_0} \times \frac{x}{\left(x^2+R^2\right)^{3/2}}$$

$$= \frac{\left(q_1+q_2\right)x}{4\pi\varepsilon_0\left(x^2+R^2\right)^{3/2}} = \frac{qx}{4\pi\varepsilon_0\left(x^2+R^2\right)^{3/2}}.$$

We note that Y-axis has been chosen so that it divides the charge distribution on the ring symmetrically.

We calculate the component of the electric field along the *Y*-axis due to charge distributions λ_1 and λ_2 separately.

First we calculate E_y^1 the electric field component along the *Y*-axis due to charge distribution λ_1. It will be

$$E_y^1 = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\lambda_1 R d\varphi}{4\pi\varepsilon_0 \left(x^2 + R^2\right)} \times \frac{R\cos\varphi}{\left(x^2 + R^2\right)^{1/2}}$$

$$= \frac{\lambda_1 R \times 2R}{4\pi\varepsilon_0 \left(x^2 + R^2\right)^{3/2}} = \frac{q_1}{2\pi^2\varepsilon_0} \frac{R}{\left(x^2 + R^2\right)^{3/2}}.$$

Similarly, we can calculate E_y^2 the electric field component along the *Y*-axis due to charge distribution λ_2. It will be

$$E_y^2 = -\frac{q_2}{2\pi^2\varepsilon_0} \frac{R}{\left(x^2 + R^2\right)^{3/2}}.$$

Therefore, E_y will be

$$E_y = \frac{q_1}{2\pi^2\varepsilon_0}\frac{R}{\left(x^2+R^2\right)^{3/2}} - \frac{q_2}{2\pi^2\varepsilon_0}\frac{R}{\left(x^2+R^2\right)^{3/2}}$$

$$= \frac{\left(q_1-q_2\right)}{2\pi^2\varepsilon_0}\frac{R}{\left(x^2+R^2\right)^{3/2}}.$$

We note that if the charge distribution along the circumference of the ring was uniform the component E_y will be zero.

By symmetry, we note that E_z is zero.