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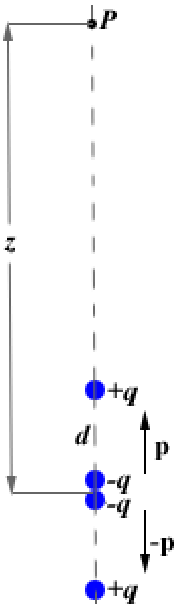
<u>Problem 28.14 (RHK)</u>

Another electric quadrupole configuration has been shown in the figure below. It consists of two dipoles whose effects at external points do not quite cancel. We have to show that the value of E on the axis of the quadrupole for points a distance z from its centre (for $z ? \; d$*) is given by*

$$E = \frac{3Q}{4\pi\varepsilon_0 z^4} \; ,$$

where $Q\left(= 2qd^2\right)$ *is called the quadrupole moment of the charge distribution.*



Solution:

By superposition principle the net electric field at P due to the four charges as shown in the figure will be

$$E_P = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{(z-d)^2} - \frac{2q}{z^2} + \frac{q}{(z+d)^2}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{z^2}\left(\left(1-\frac{d}{z}\right)^{-2} - 2 + \left(1+\frac{d}{z}\right)^{-2}\right).$$

As at the point *P*, we can assume

$\frac{d}{z} = 1$, we use the binomial expansion up to the second order for finding the electric field as in the first order the field is zero. That is we use

$$(1+\xi)^n \; ; \; 1 + n\xi + \frac{n(n-1)}{2}\xi^2 + O(\xi^3).$$

We find

$$E_P = \frac{1}{4\pi\varepsilon_0}\frac{q}{z^2}\begin{pmatrix} 1+\frac{2d}{z}+\frac{(-2)(-3)}{2}\frac{d^2}{z^2}-2 \\ +1-\frac{2d}{z}+\frac{(-2)(-3)}{2}\frac{d^2}{z^2} \end{pmatrix}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{z^4}\times 6d^2$$

$$= \frac{3Q}{4\pi\varepsilon_0 z^4},$$

where we have defined quadrupole moment of the charge combination as

$$Q = 2qd^2.$$