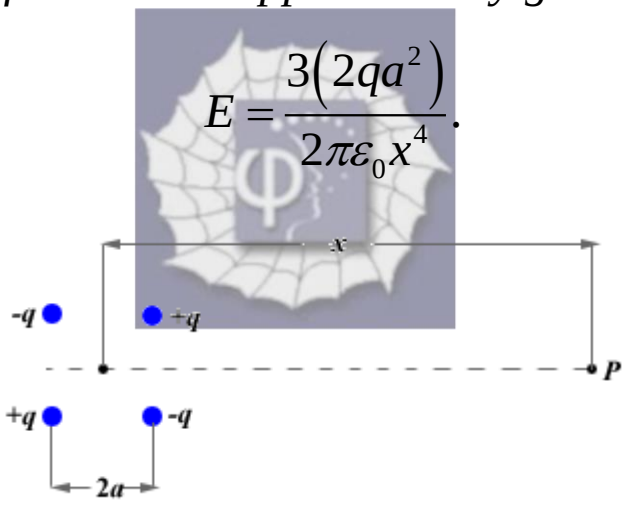
293.

Problem 28.13 (RHK)

One type of electric quadrupole is formed by four charges located at the vertices of a square of side 2a. Point P lies a distance x from the centre of the quadrupole on a line parallel to two sides of the square as shown in the figure. For $x ? a$, we have to show that the electric field at P is approximately given by

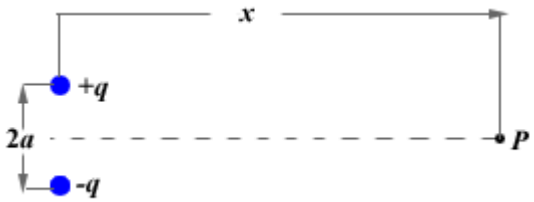
$$E = \frac{3(2qa^2)}{2\pi\varepsilon_0 x^4}.$$



Solution:

A quadrupole is a combination of four charges of equal magnitude with two of opposite signs such that the total charge is zero. A quadrupole configuration is shown in the figure. It can be considered as a combination of two dipoles. We recall that the field at a distant point from a dipole at the location as shown in the figure below is

$$E = \frac{1}{4\pi\varepsilon_0} \times \frac{p}{x^3},$$



for x ? $2a$, and $p = 2aq$.

The net electric field at P in the configuration of the quadrupole shown in the figure can be found as a vector sum of electric fields due to the two dipoles. It is given by

$$E_P = \frac{1}{4\pi\varepsilon_0} \frac{p}{(x-a)^3} - \frac{1}{4\pi\varepsilon_0} \frac{p}{(x+a)^3}.$$

For x ? a, we approximate

$$\left(1+\frac{a}{x}\right)^n ; \; 1+n\frac{a}{x}.$$

In this approximation

$$E_P = \frac{1}{4\pi\varepsilon_0} \frac{p}{x^3}\left(\frac{6a}{x}\right).$$

As $p = 2qa$, we find

$$E_P = \frac{3\left(2qa^2\right)}{2\pi\varepsilon_0 x^4}.$$