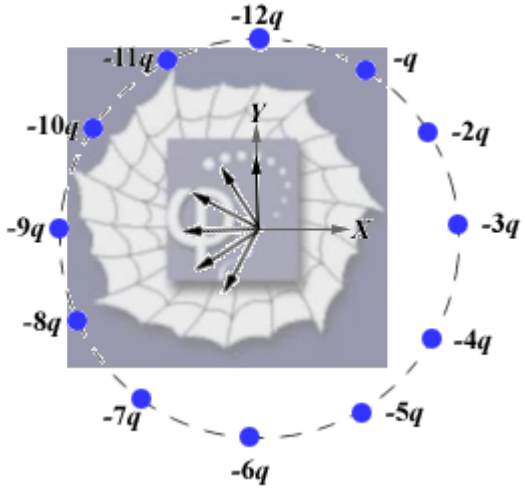
291.

Problem 28.9 (RHK)

A clock face has negative point charges $-q$, $-2q$, $-3q, \ldots, -12q$ *fixed at the positions of the corresponding numerals. The clock hands do not perturb the field. We have to find the hour hand points in the same direction as the electric field at the centre of the dial.*



Solution:

By the superposition principle the net electric field at the centre of the clock due to the 12 charges will be the vector sum of the fields at the centre due to each charge. As the direction of the electric field is along the line joining the point where the field is being calculated to the charge, we calculate the contribution to the field by the

diametrically opposite charges. It may be noted that the magnitude of the electric field due to each pair of diametrically opposite charges will be the same but their directions will be different as shown in the figure. The magnitude of the electric field at the centre due to each pair of diametrically opposite charges will be

$$E_{pair} = \frac{6q}{4\pi\varepsilon_0 r^2},$$

where r is the radius of the clock.

From the diagram where electric filed vectors have been shown, we note that the X- component of the net electric field at the centre of the clock will be

$$E_x = -\frac{(6q)}{4\pi\varepsilon_0 r^2}\left(2\cos\frac{\pi}{6} + 2\cos\frac{\pi}{3} + 1\right) = -\frac{(6q)}{4\pi\varepsilon_0 r^2}(3.732).$$

And the Y-component of the electric field will be

$$E_y = \frac{6q}{4\pi\varepsilon_0 r^2}.$$

Angle of the electric field vector from the X-axis will be

$$\tan\theta = \frac{E_y}{E_x} = -\frac{1}{3.732} = -0.2679.$$

And the direction of the field as measured from the -X-axis will be

$$\phi = \tan^{-1}\left(\frac{1}{3.732}\right) = 15^0.$$

It is 15^0 beyond 9'O clock. Therefore, the hour hand will be pointing at 9.30.