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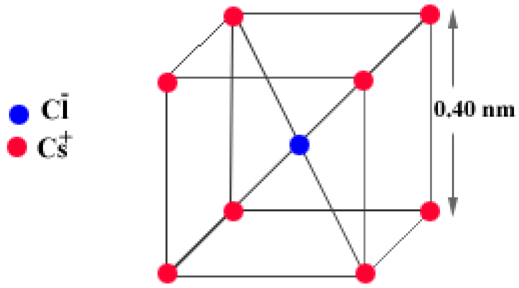
Problem 22.35P (HRW)

In the basic CsCl (caesium chloride) *crystal structure,* Cs^+ *ions form the corners of a cube and a* Cl^- *ion is at the cube's centre*. *The edge length of the cube is* 0.40 nm. *The* Cs^+ *ions are each deficient by one electron* (and thus each has a charge of $+e$), *and the* Cl^- *ion has one excess electron* (and thus has a charge of $-e$). (a) *We have to find the magnitude of the net electrostatic force on the* Cl^- *ion by the eight* Cs^+ *ions at the corners of the cube.* (b) *If one of the* Cs^+ *ions is missing the crystal is said to have a defect. We have to find the magnitude of the net electrostatic force exerted on the* Cl^- *ion by the remaining* Cs^+ *ions.*



Solution:

(a)

We not that relative to the Cl^- ion at the centre of the cubic crystal there are a pair of Cs^+ ions which exert equal and opposite electrostatic forces on the Cl^- ion. Therefore, the magnitude of the net electrostatic force on the Cl^- ion due to the eight Cs^+ ions located at the vertices of the cubic lattice will be zero.

(b)

When one of the Cs^+ ions is missing there will be an imbalance of force on the Cl^- ion. The magnitude of the force on the Cl^- ion due to the remaining seven Cs^+ ions will therefore be equal to the force that the missing Cs^+ ion would have exerted on the Cl^- ion.

It is given that the edge length of the cube is 0.40 nm. Therefore, the distance of the centre of the cube from any of the vertices of the cube will be

$$d = \frac{\sqrt{3}}{2} a = \frac{\sqrt{3}}{2} \times 0.40 \times 10^{-9} \text{ m} = 3.46 \times 10^{-10} \text{ m}.$$

By Coulomb's law we calculate the magnitude of the force exerted by two charges e at a distance d from each other. Therefore, the magnitude of the force on the Cl^- ion due to the remaining seven Cs^+ ions will be

$$F = \frac{e^2}{4\pi\varepsilon_0 d^2} = \frac{8.99\times10^9\times\left(1.6\times10^{-19}\right)^2}{\frac{3}{4}\left(0.40\times10^{-9}\right)^2}\ \text{N} = 1.9\times10^{-9}\ \text{N}.$$