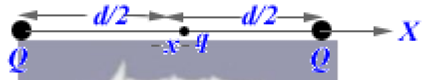
282.

Problem 27.23 (RHK)

Two positive charges +Q are held fixed a distance d apart. A particle of negative charge –q and mass m is placed midway between them and displaced along the line joining the charges.

We have to calculate the period of oscillation.



Solution:

Two positive charges +*Q* are held fixed at $-d/2$ and $+d/2$ from the origin of *X*-axis as shown in the figure. A charge *q* is placed at the origin which is midway between the two charges +*Q* and is displaced along the *X*-axis by a small distance *x*. We assume

$$x = \; d \; .$$

The net Coulomb force on the charge *q* will be

$$\vec{F}(x) = \frac{Qq}{4\pi\varepsilon_0}\left(\frac{1}{\left(\frac{d}{2}+x\right)^2} - \frac{1}{\left(\frac{d}{2}-x\right)^2}\right)\times\hat{x}$$

$$= \frac{Qq}{\pi\varepsilon_0 d^2}\left(\frac{1}{\left(1+\frac{2x}{d}\right)^2} - \frac{1}{\left(1-\frac{2x}{d}\right)^2}\right)\times\hat{x}$$

$$\simeq -\frac{8Qqx}{\pi\varepsilon_0 d^3}\hat{x}.$$

Equation of motion of the charge q of mass m will therefore be

$$m\frac{d^2\vec{x}}{dt^2} = \vec{F}(x) = -\frac{8Qqx}{\pi\varepsilon_0 d^3}\hat{x}.$$

Scalar form of this equation is

$$m\frac{d^2x}{dt^2} + \frac{8Qq\,x}{\pi\varepsilon_0 d^3} = 0\ .$$

It is of the form of equation of simple harmonic motion (SHM). The period of SHM described by this equation will be

$$\frac{2\pi}{T} = \left(\frac{8Qq}{\pi\varepsilon_0 m d^3}\right)^{1/2}\ .$$

Or

$$T=\left(\frac{\varepsilon_0\pi^3 m d^3}{2Qq}\right)^{1/2}.$$