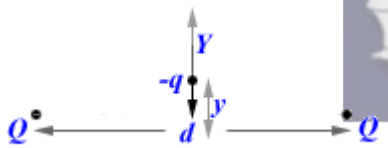
281.

Problem 27.22 (RHK)

Two positive charges Q are held fixed a distance d apart. A particle of negative charge –q and mass m is placed midway between them and then given a small displacement perpendicular to the line joining them and released. We have to show that the particle describes simple harmonic motion of period $\left(\varepsilon_0 m\pi^3 d^3 / qQ\right)^{1/2}$.

Solution:



It is given that two positive charges +*Q* are held fixed a distance *d* apart. A particle of negative charge –*q* and mass m is placed midway between them and given a small displacement perpendicular to the line joining them and is released. Let the displacement be *y* along the *Y*-axis. As the displacement is small we have the condition

$$y = d \ .$$

By symmetry the net Coulomb force on the charge $-q$ due to the charges Q will be in the direction $-\hat{y}$ and its magnitude will be

$$F(y) = \frac{2Qq}{4\pi\varepsilon_0\left(\frac{d^2}{4}+y^2\right)} \times \frac{y}{\left(\frac{d^2}{4}+y^2\right)^{1/2}}$$

$$= \frac{2Qqy}{4\pi\varepsilon_0\left(\frac{d^2}{4}+y^2\right)^{3/2}} .$$

In the approximation $y = d$, we approximate F as

$$F(y) ; \frac{8Qqy}{2\pi\varepsilon_0 d^3}.$$

Equation of motion of the charge $-q$ that has mass m will therefore be

$$m\frac{d^2\mathrm{r}y}{dt^2} = -\frac{8Qqy}{2\pi\varepsilon_0 d^3}\hat{y}.$$

Scalar form of this equation is

$$m\frac{d^2y}{dt^2} + \frac{8Qq}{(2\pi\varepsilon_0)d^3}y = 0.$$

It is an equation of simple harmonic motion form. The period of SHM of the motion is

$$\frac{2\pi}{T} = \left(\frac{8Qq}{2\pi\varepsilon_0 m d^3} \right)^{1/2}.$$

Or

$$T = \left(\frac{\varepsilon_0 m \pi^3 d^3}{Qq} \right)^{1/2}.$$