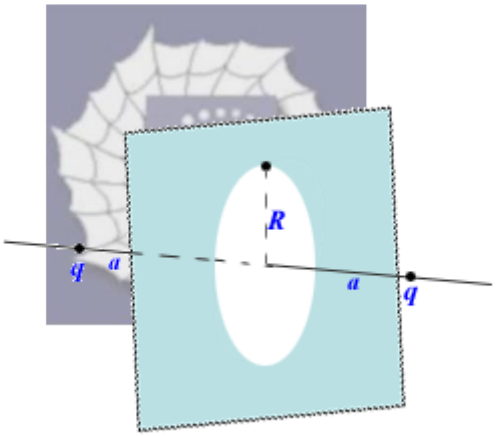
279.

Problem 27.19 (RHK)

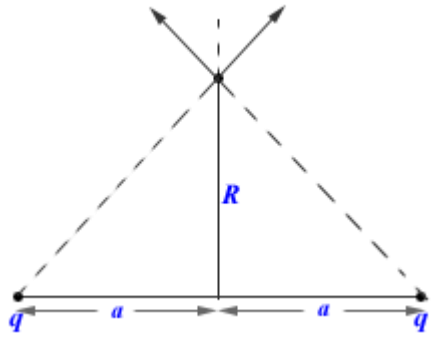
Two equal positive point charges q are held a fixed distance 2a apart. A point test charge is located in a plane that is normal to the line joining these charges and midway between them. We have to find the radius R of the circle in this plane for which the force on the test particle has a maximum value.



Solution:

It is given that two equal point charges q are held a fixed distance $2a$ apart. A point test charge is located in a plane that is normal to the line joining the two charges and midway between them. We have to find the radius R of the circle in this plane for which the force on the test charge has a maximum possible value. In the following

diagram we have shown the forces acting on the test charge due to the two charges.



By symmetry the net force on the test charge will be in the vertical direction. The horizontal components of the Coulomb force on the test charge due to the two charges will cancel each other. Therefore, the magnitude of the net force on the test charge will be

$$F(R)=\frac{2}{4\pi\varepsilon_0}\times\frac{q\times 1}{\left(a^2+R^2\right)}\times\frac{R}{\left(a^2+R^2\right)^{1/2}}\ .$$

Condition for maximum of $F(R)$ is

$$\frac{dF(R)}{dR}=0\ .$$

Or

$$\frac{2q}{4\pi\varepsilon_0}\left(\frac{1}{\left(a^2+R^2\right)^{3/2}}-\frac{3}{2}\times\frac{R\times 2R}{\left(a^2+R^2\right)^{5/2}}\right)=0,$$

or

$$\left(a^2+R_{\max}{}^2\right)-3R_{\max}{}^2=0\ .$$

Therefore,

$$R_{\max} = \frac{a}{\sqrt{2}} \ .$$