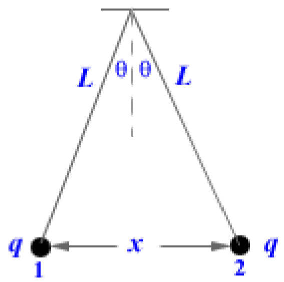
277.

Problem 27.16 (RHK)

Two similar tiny balls of mass m are hung from silk threads of length L and carry equal charges q (as shown in the figure). *We may assume that* θ *is so small that* $\tan\theta$ *can be replaced by its approximate equal,* $\sin\theta$.
(a) *We have to show that in this approximation at equilibrium*

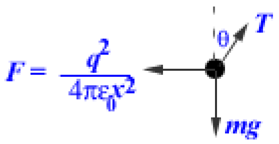
$$x = \left(\frac{q^2 L}{2\pi\varepsilon_0 mg} \right)^{1/3} ,$$

where x *is the separation between the balls.* (b) *If* $L = 122$ cm, $m = 11.2$ g, and $x = 4.70$ cm, *we have to find the value of q.*
(c) *Assuming that both balls are conducting and one is discharged, we have to explain what will happen to the two conducting balls in this situation. We have to find the new equilibrium separation.*



Solution:

Let us draw the free-body-diagram of the charge 1. It will be as shown in the figure below.



Forces acting on charge 1 are weight mg acting vertically downward, force of Coulomb repulsion $F = \dfrac{q^2}{4\pi\varepsilon_0 x^2}$ acting horizontally as shown, and the tension in the string T acting at angle θ along the string. As the charge is in equilibrium, sums of the components of the forces in the vertical direction and of the components of the forces in the horizontal direction have to be zero. Therefore, we have the following two equations as each charge is in equilibrium.

$$T\cos\theta = mg,$$

$$T\sin\theta = \frac{q^2}{4\pi\varepsilon_0 x^2} \ .$$

In the approximation $\tan\theta ; \sin\theta$, we therefore have the relation

$$\sin\theta = \frac{q^2}{4\pi\varepsilon_0 x^2 mg} \ .$$

From the geometry of the problem, we note that

$$\sin\theta = \frac{x}{2L} \ .$$

We thus have the relation

$$\frac{x}{2L} = \frac{q^2}{4\pi\varepsilon_0 x^2 mg} \ ,$$

Or

$$x^3 = \frac{q^2 L}{2\pi\varepsilon_0 mg} \ .$$

Therefore,

$$x = \left(\frac{q^2 L}{2\pi\varepsilon_0 mg} \right)^{1/3} \ .$$

(b)

We use the data

$L = 122$ cm $= 1.22$ m, $m = 11.2 \times 10^{-3}$ kg, and $x = 4.70 \times 10^{-2}$ m, for calculating the value of q.

We have the equation

$$q^2 = \frac{x^3\left(4\pi\varepsilon_0\right)mg}{2L} = \frac{\left(4.7\times10^{-2}\right)^3\times11.2\times10^{-3}\times9.81}{2\times1.22\times8.99\times10^9}\ \text{C}^2$$

$$=520\times10^{-18}\ \text{C}^2.$$

And

$q = 22.8\times10^{-9}$ C.

(c)

Now if one of the balls is discharged there will be no Coulomb force on each due to the other and because of the tension in the strings they will move toward each other like pendulums. When they come in contact with each other they will share charge and the charge on each conducting sphere will become $q' = q/2 = 11.4\times10^{-9}$ C. The new equilibrium separation can be found from the relation

$$\frac{x'}{x} = \left(\frac{q'}{q}\right)^{2/3},$$

or

$$x' = 4.70\times\left(\frac{1}{2}\right)^{2/3}\ \text{cm} = 2.96\ \text{cm}.$$