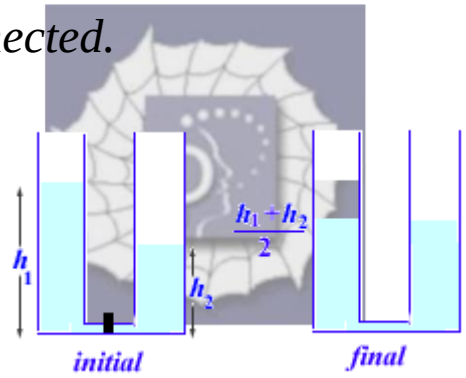
77 (b).

Problem 17.23 (RHK)

Two identical cylindrical vessels with their bases at the same level each contain a liquid of density ρ. The area of either base is A, but in one vessel the liquid height is h_1 and in the other h_2. We have to find the work done by gravity in equalizing the levels when the two vessels are connected.



Solution:

Problem has been described through the diagram. In the initial situation liquid in the left vessel is filled to height h_1 and the height of liquid in the right vessel is h_2. When the tap joining the two vessels is removed, liquid will flow from the container that has higher liquid level to the other. We assume that there is sufficient friction so that the flow speed is nearly zero. The flow will stop when

liquid levels in both containers equalise and attain equal height, $(h_1 + h_2)/2$. We will find the work done by gravity in the changing the system from the initial to the final situation shown in the diagram by computing the change in potential energy of the system.

Potential energy of a liquid of density ρ contained in a cylindrical vessel of cross-section *A* up to height *h* from the base is

$$PE = \rho g A \int_0^h h dh = \frac{1}{2} \rho g A h^2.$$

Using this result we find that the potential energy of the system in the initial situation is

$$PE_{initial} = \frac{1}{2} \rho g A \left(h_1^{\,2} + h_2^{\,2} \right).$$

Potential energy of the system in the final situation is

$$PE_{final} = 2 \times \frac{1}{2} \rho g A \left(\frac{h_1 + h_2}{2} \right)^2 = \frac{1}{4} \rho g A \left(h_1 + h_2 \right)^2.$$

Therefore, the work done by gravity in equalising the levels is

$$W = PE_{initial} - PE_{final} = \frac{1}{4} \rho g A \left(h_1 - h_2 \right)^2.$$