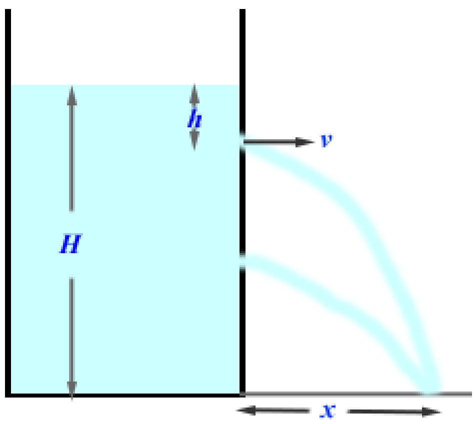
77 (a).

Problem 15.75P (HRW)

A tank is filled with water to a height H. A hole is punched in one of the walls at a depth h below the water surface. We have to show (a) *that the distance x from the base of the tank to the point at which the resulting stream strikes the floor is given by* $x = 2\sqrt{h(H-h)}$. (b) *We have to find whether another hole can be punched at another depth to produce a second stream that would have the same range.* (c) *We have to find the depth at which a hole will give the maximum distance from the base of the tank.*



Solution:

Speed with which liquid flows out at depth *h* is $v = \sqrt{2gh}$.

Time *t* taken by the stream for free-fall a height *H-h* vertically is given by

$$H - h = \tfrac{1}{2} g t^2 ,$$

or

$$t = \sqrt{\frac{2(H-h)}{g}}.$$

Horizontal distance *x* traversed by stream of liquid in time *t* is $x = vt = \sqrt{2gh} \times \sqrt{\frac{2(H-h)}{g}} = 2\sqrt{(H-h)h}$.

As this function is symmetric in $(H-h)$ and *h*, stream coming out of another hole at a depth *H-h* from the top of the liquid level in the tank will also have the same range *x*.

We next find *h* for which will give maximum range *x*. This will be given by the solution of the equation

$$\frac{dx}{dh} = 0.$$

That is

$$2 \times \left(\tfrac{1}{2} (H - h)^{-1/2} \right) (H - 2h) = 0.$$

Its solution is

$$h = \frac{H}{2}.$$

Stream from a hole at depth H/2 will have the maximum horizontal range.