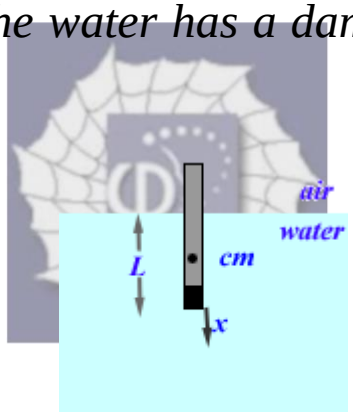
70 (b).

Problem 17.46 (RHK)

A cylindrical wooden log is loaded with lead at one end so that it floats upright in water. The length of the submerged portion is L=2.56 m. *The log is set into vertical oscillation. We have to show* (a) *that the motion is simple harmonic.* (b) *Find the period of oscillation. We can neglect that the water has a dampening effect on the motion.*



Solution:

Let the cross-sectional area of the rod be A m^2. Length of the submerged portion of the rod, L, is 2.56 m. Mass of the rod, M, will be equal to the mass of the water displaced by it. That is

$M = 2.56 \times 1.0 \times 10^3 A$ kg.

When the rod is pushed down by x m, it will be pushed up by buoyant force equal to the weight of the water displaced. The upward force F will act in the direction opposite to the displacement of the rod,

$$F = -1.0 \times 10^3 \times 9.8 \times Ax \text{ N.}$$

By using the second law of motion, taking care of the direction of acceleration and that of force, equation of motion of the centre of mass (cm) of the rod can be written as

$$M \frac{d^2x}{dt^2} = F,$$

or

$$A \times 2.56 \times 1.0 \times 10^3 \frac{d^2x}{dt^2} = -A \times 1.0 \times 10^3 \times 9.8x$$

or

$$\frac{d^2x}{dt^2} + \frac{9.8}{2.56} x = 0.$$

It is the equation of harmonic motion. Therefore, motion is oscillatory. Period of oscillation T is

$$T = \frac{2\pi}{\sqrt{\frac{9.8}{2.56}}} \text{ s } = 3.21 \text{ s.}$$