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Problem 3.43 (R)

Consider electromagnetic radiation to consist of photons, that is, particles of zero rest mass and of energy $E = h\nu$. *We have to show that the Doppler shift can be obtained from the transformation laws for the components of momentum and the energy.*

Solution:

Let us consider two inertial frames S and S' .Let S'be moving with respect to S in the direction of x-axis to the right with speed V. Let a source of light stationary in S emit photons of frequency ν. An observer in S'measure frequency of photons and find it to ν'. With respect to observer in $S'$the source is receding away from S with speed V.

Energy of a photon of frequency ν is $E = h\nu$ and its momentum $p = h\nu/c$.

Energy of this photon as measured by S' will be $E' = h\nu'$ and its momentum $p' = h\nu'/c$.

The Lorentz transformation of energy and momentum is

$$E' = \frac{E - pV}{\sqrt{1 - V^2/c^2}}.$$

Therefore, we have

$$h\nu' = \frac{h\nu - h\nu V/c}{\sqrt{1 - V^2/c^2}}, \nu$$

or

$$\nu' = \frac{\nu(1 - V/c)}{\sqrt{1 - V^2/c^2}}.$$

And when the source and the observer are moving toward each other with speed V, we have

$$\nu' = \frac{\nu(1 + V/c)}{\sqrt{1 - V^2/c^2}}.$$

The relations between the emitted frequency ν and the observed frequency ν' when the source and the observer have constant relative velocity derived above are the relativistic Doppler effect.