194.

Problem 3.41 (R)

An electron moves in the positive x-direction in frame S at a speed v = 0.8 *c*. (a) *We have to find its momentum and its energy in frame S.* (b) *Consider a frame* S' *moving to the right of S with a speed* 0.6 *c. We have to find the momentum and energy of the electron in the frame* S'.

Solution:

(a)

We will solve this problem by considering that under Lorentz transformations E/c transforms like *ct* and *p* transforms like *x*. If in the frame *S* the energy and momentum are *E* and *p*, then in the frame S', which is moving with speed *v* to the right with respect to *S*, values of energy and momentum E' and p' will be

$$p' = \frac{p - vE/c^2}{\sqrt{1 - v^2/c^2}},$$

$$E' = \frac{E - pv}{\sqrt{1 - v^2/c^2}}.$$

In the frame *S* electron is observed to be moving with speed $u = 0.8\,c$ in the x-direction. Its momentum will therefore be

$$p = \frac{m_e u}{\sqrt{1-u^2/c^2}} = \frac{0.8\times m_e c}{\sqrt{1-0.64}} = 1.33\times m_e c,$$
$$= \frac{0.8\times 9.11\times 10^{-31}\times 3\times 10^8}{0.6}\ \text{kg m s}^{-1},$$
$$= 3.64\times 10^{-22}\ \text{kg m s}^{-1}.$$

Energy of the electron in the frame *S* will be

$$E = \frac{m_e c^2}{\sqrt{1-u^2/c^2}} = 1.66\times m_e c^2 = \frac{0.51}{0.6}\ \text{MeV} = 0.85\ \text{MeV}.$$

(b)

Using the Lorentz transformations for energy and momentum, we will determine their values in S', which is moving to the right with speed $v = 0.6\,c$ with respect to *S*. We have

$$p' = \frac{p - Ev/c^2}{\sqrt{1 - v^2/c^2}} = \frac{(1.33 - 1.66 \times 0.6) \times m_e c}{0.8} = 0.417 \times m_e c,$$

$$= 1.14 \times 10^{-22} \text{ kg m s}^{-1}.$$

And energy E' will be

$$E' = \frac{E - pv}{\sqrt{1 - v^2/c^2}} = \frac{(1.66 - 1.33 \times 0.6)}{\sqrt{1 - (0.6)^2}} \times m_e c^2 = 1.077 \times m_e c^2,$$

$$= 0.549 \text{ MeV}.$$