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Problem 3.36 (R)

A charged π*-meson* (*rest mass* $m_\pi = 273\,\mathrm{m}_e$) *at rest decays into a neutrino (zero rest mass) and a muon* (*rest mass* $m_\mu = 207\,\mathrm{m}_e$). *We have to find the kinetic energy of the neutrino and that of the muon.*

Solution:

From conservation of momentum we note as we are considering the decay of a π-meson at rest, the magnitude of the two decayed particles, the neutrino and the muon, will be equal and that they will be moving in opposite directions. Let the magnitude of momentum of either of the particles be *p*. Let the energy of the neutrino be E_ν and that of the muon be E_μ. Applying conservation of energy, we have

$$m_\pi c^2 = E_\nu + E_\mu .$$

$$E_\nu = pc,$$

$$E_\mu = \sqrt{p^2c^2 + m_\mu{}^2c^4}\ .$$

So, we have

$$\left(p^2c^2 + m_\mu{}^2c^4\right) = m_\pi{}^2c^4 + p^2c^2 - 2m_\pi c^2 \times \left(pc\right).$$

Simplifying this algebraic equation, we find the kinetic energy of the neutrino which is equal to its energy as it has zero rest mass,

$$pc = \frac{1}{2} \times m_\pi c^2 \times \left(1 - \left(\frac{m_\mu}{m_\pi}\right)^2\right)$$

$$= \frac{273}{2} \times m_e c^2 \left(1 - \left(\frac{207}{273}\right)^2\right) = 58.02 \times m_e c^2 = 58.02 \times 0.51 \text{ MeV},$$

$$= 29.59 \text{ MeV}.$$

Kinetic energy of the muon will be

$$KE_\mu = m_\pi c^2 - E_\nu - m_\mu c^2 = \left(273 - 58.02 - 207\right) \times m_e c^2,$$

$$= 7.98 \times 0.51 \text{ MeV} = 4.07 \text{ MeV}.$$