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Problem 2.23 (R)

The radius of our galaxy is 3×10^{20} m, *or about* 3×10^{4} light-years. *We have to answer* (a) *whether, in principle, a person can travel to the edge of our galaxy in a normal lifetime;* (b) *and calculate the constant velocity needed to make the trip in* 30 years (*proper time*).

Solution:

(a)

If the spaceship carrying the person to the edge of the galaxy travels with speed very close to the speed of light, although the time period measured by the observer on the Earth will far exceed the normal life span yet the proper time that is the time shown on the watch of the traveller can be a normal period comparable to the lifetime that is about 30 years or so. So the answer to the first part of the problem is in affirmative.

(b)

We will now calculate the speed with which the spaceship should travel with respect to the Earth so that the proper time of 30 years is measured as 3×10^4 years by an observer on the Earth. Let the speed of the spaceship with respect to the Earth be *v*. We will find v from the time dilatation relation

$$t = \frac{t_0}{\sqrt{1-\beta^2}},$$

where $\beta = v/c$. We therefore have

$$\sqrt{1-\beta^2} = \frac{30}{3\times10^4},$$

or

$$1\text{-}\beta^2 = 10^{-6},$$

or

$$\beta = \left(1-0.000001\right)^{1/2} = \left(0.999999\right)^{1/2} = 0.9999995,$$

or

$$v = 0.9999995\ c.$$