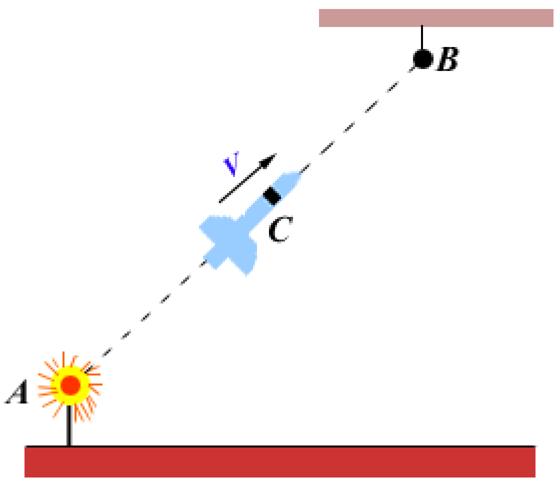
181.

<u>Problem 2.44 (R)</u>

A, on the Earth, is sending signals with a flashlight every six minutes. B is on a space station that is stationary with respect to the Earth. C is on a rocket travelling from A to B with a constant velocity of 0.6c relative to A. (a) *We have to find the time interval between the signals received by B from A;* (b) *the time interval between the signals received by C from A; and* (c) *if C flashes a light using intervals equal to those he received from A then the interval between the signals received by B from C.*



Solution:

For solving this problem we have to use the longitudinal Doppler effect in relativity.

Let the source frequency be ν_0. If the source and the observer move away from one another with velocity *V*, then the observed frequency ν is related to ν_0 and the relative velocity *V* as

$$\nu = \nu_0 \sqrt{\frac{c-V}{c+V}}.$$

As period τ is the reciprocal of the frequency, ν, we have

$$\tau = \tau_0 \sqrt{\frac{c+V}{c-V}}.$$

If the source and the observer move toward one another with velocity *V*, the Doppler shifted frequency and period are related to the transmitting frequency, ν_0, and period, τ_0, as

$$\nu = \nu_0 \sqrt{\frac{c+V}{c-V}},$$

and

$$\tau = \tau_0 \sqrt{\frac{c-V}{c+V}}.$$

(a)

It is given that the space station *B* is stationary with respect to the Earth, *A*. Therefore, there will not be any Doppler shift between the signals transmitted from *A* and received by *B*. If the signals are transmitted by *A* every six minutes, $\tau_0 = 6\text{ min}$. The period of signals received by *B* will also be 6 min.

(b)

As the source *A* and the receiver *C* are moving away from one another with velocity 0.6 *c*, the period of signals received by *C* transmitted from *A* with period $\tau_0 = 6\text{ min}$ will be Doppler shifted. It will be

$$\tau_C = 6 \times \sqrt{\frac{c+0.6c}{c-0.6c}}\text{ min} = 12\text{ min}.$$

(c)

As the source C and the observer B are moving toward one another with velocity $V = 0.6\,c$ the period of signals transmitted by C, τ_C, and received by B will be Doppler shifted. We have

$$\tau_B = \tau_C \sqrt{\frac{c-V}{c+V}} = 12 \times \sqrt{\frac{0.4}{1.6}}\ \text{min} = 6\,\text{min}.$$