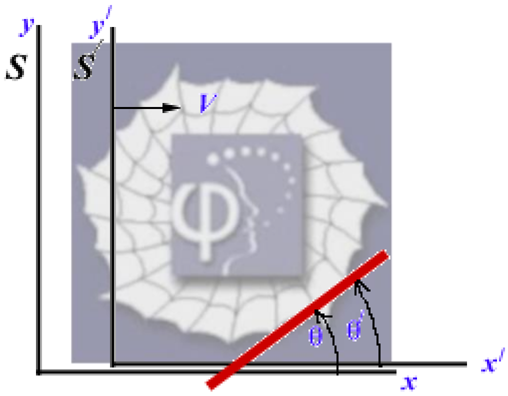
Problem 2.40 (R)

A stick at rest in S has a length L and is inclined at angle θ to the x-axis. We have to find its length L' and angle of inclination θ' to the x' axis as measured by an observer in S' moving with a speed v relative to x along the $x - x'$ axes.



Solution:

In the inertial frame S the rod is at rest. Its length measured in S is L. The x and y components of the length of the rod in frame S are

$$L_x = L\cos\theta \text{ and } L_y = L\sin\theta,$$

where as shown in the diagram the rod is inclined at angle θ with respect to x-axis.

The inertial frame S' is moving with speed V in the direction of x-axis as measured by S. Let the length of the rod measured by S' be L' and the inclination of the rod with respect to x' axis be by an angle θ'. Its x' and y' components measured by S' will be

$$L'_{x'} = L'\cos\theta' \text{ and } L'_{y'} = L'\sin\theta'.$$

By Lorentz transformations we note that

$$L'_{x'} = L_x\sqrt{1 - V^2/c^2} \text{ and } L'_{y'} = L_y.$$

The length L' will therefore be

$$L' = \left(L'^{\,2}_{x'} + L'^{\,2}_{y'}\right)^{1/2} = \left(L^2\cos^2\theta\left(1-\beta^2\right) + L^2\sin^2\theta\right)^{1/2},$$

or

$$L' = L\left(1-\beta^2\cos^2\theta\right)^{1/2},$$

where

$$\beta = \frac{V}{c}.$$

And

$$\frac{L'_{y'}}{L'_{x'}} = \frac{L_y}{L_x\sqrt{1-\beta^2}} = \frac{\tan\theta}{\sqrt{1-\beta^2}}.$$