179.

Problem 2.24 (R)

Two spaceships, each of proper length 100 m, *pass near one another heading in opposite directions. If an astronaut at the front one ship measures a time interval of* 2.50×10^{-6} s *for the second ship to pass him, then* (a) *we have to find the relative velocity of the spaceships;* (b) *the time interval measured on the first spaceship for the front of the second spaceship to pass from the from the front to the back of the first spaceship.*

Solution:

Observations on the motion of the second spaceship are being made from the first spaceship. Let the speed of the second spaceship as observed from the first spaceship be v m s^{-1}.

The proper length of each spaceship is 100 m. The length of the second spaceship measured by the first spaceship will be Lorentz contracted

$$l = 100\sqrt{1-v^2/c^2}\ \text{m}.$$

The time for the second spaceship to pass the first spaceship is observed to be 2.50×10^{-6} s. This will be equal to l/v . Thus we have

$$\frac{100\times\sqrt{1-v^2/c^2}}{v}=2.50\times10^{-6}.$$

We have

$$\sqrt{1-v^2/c^2}=2.50\times10^{-8}\times3\times10^{8}\left(v/c\right),$$

or

$$1-\frac{v^2}{c^2}=56.25\times\frac{v^2}{c^2},$$

or

$$\frac{v}{c}=\sqrt{\frac{1}{57.25}}=0.132.$$

And the relative velocity of the two spaceships is

$$v=0.132\times3\times10^{8}\ \text{m s}^{-1}=3.96\times10^{7}\ \text{m s}^{-1}.$$

(b)

The time interval measured on the first spaceship for the front of the second spaceship to pass from the front to the back of the first spaceship will be its length divided by the speed with which the second spaceship is moving towards it. The length of the spaceship is 100 m.

Therefore, the time taken will be

$$100/3.96\times10^{7}\ \text{s} = 2.52\times10^{-6}\ \text{s}.$$