

174.

Problem 21.54 (RHK)

(a) *Suppose we have a particle accelerated from rest by the action of a force F . Assuming that Newton's second law for a particle, $F = \frac{dp}{dt}$, is valid in relativity.*

We have to show that the final kinetic energy K can be written, using the work-energy theorem, as $K = \int_0^v v dp$.

(b) *By substituting the expression for relativistic momentum and carrying out the integration, we have to show that*

$$K = \frac{mc^2}{\sqrt{1 - v^2/c^2}} - mc^2.$$

Solution:

(a)

For simplicity we will consider one dimensional motion.

According to Newton's second law

$$F = \frac{dp}{dt},$$

where p is the momentum of the particle at time t . When the particle moves by distance dx in the time interval dt , then according to the work-energy theorem the change in the kinetic energy of the particle will be

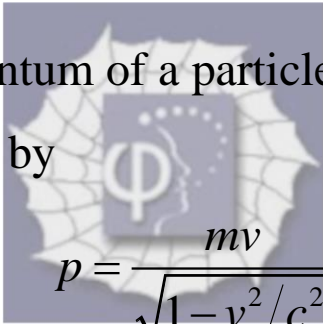
$$dK = Fdx = \frac{dp}{dt} dx = \frac{dp}{dt} v dt = v dp,$$

or

$$K = \int_0^v v dp .$$

(b)

Relativistic momentum of a particle of mass m and velocity v is given by



$$p = \frac{mv}{\sqrt{1 - v^2/c^2}} .$$

Therefore,

$$dp = \frac{m dv}{\left(1 - \frac{v^2}{c^2}\right)^{3/2}} .$$

We, therefore, have

$$K = \int_0^v \frac{m v dv}{\left(1 - \frac{v^2}{c^2}\right)^{3/2}} .$$

Let us make the substitution in the integration variable

$$1 - \frac{v^2}{c^2} = \xi.$$

This gives

$$-\frac{2v dv}{c^2} = d\xi.$$

And, we have

$$K = -\frac{mc^2}{2} \int_0^{1-\frac{v^2}{c^2}} \frac{d\xi}{\xi^{3/2}} = mc^2 \left[\frac{1}{\sqrt{1-v^2/c^2}} - 1 \right].$$

The energy of a particle of mass m moving with speed v will therefore be

$$E = K + mc^2 = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}}.$$