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<u>Problem 21.54 (RHK)</u>

(a) *Suppose we have a particle accelerated from rest by the action of a force F. Assuming that Newton's second law for a particle,* $F = \dfrac{dp}{dt}$*, is valid in relativity. We have to show that the final kinetic energy K can be written, using the work-energy theorem, as* $K = \int_0^v v\,dp$.

(b) *By substituting the expression for relativistic momentum and carrying out the integration, we have to show that*

$$K = \frac{mc^2}{\sqrt{1 - v^2/c^2}} - mc^2.$$

<u>Solution:</u>

(a)

For simplicity we will consider one dimensional motion. According to Newton's second law

$$F = \frac{dp}{dt},$$

where p is the momentum of the particle at time t. When the particle moves by distance dx in the time interval dt, then according to the work-energy theorem the change in the kinetic energy of the particle will be

$$dK = Fdx = \frac{dp}{dt}dx = \frac{dp}{dt}vdt = vdp,$$

or

$$K = \int_0^v vdp \ .$$

(b)

Relativistic momentum of a particle of mass m and velocity v is given by

$$p = \frac{mv}{\sqrt{1 - v^2/c^2}}.$$

Therefore,

$$dp = \frac{mdv}{\left(1 - \frac{v^2}{c^2}\right)^{3/2}}.$$

We, therefore, have

$$K = \int_0^v \frac{mv\,dv}{\left(1 - \frac{v^2}{c^2}\right)^{3/2}}.$$

Let us make the substitution in the integration variable

$$1-\frac{v^2}{c^2}=\xi.$$

This gives

$$-\frac{2v\,dv}{c^2}=d\xi.$$

And, we have

$$K=-\frac{mc^2}{2}\int_0^{1-\frac{v^2}{c^2}}\frac{d\xi}{\xi^{3/2}}=mc^2\left[\frac{1}{\sqrt{1-v^2/c^2}}-1\right].$$

The energy of a particle of mass *m* moving with speed *v* will therefore be

$$E=K+mc^2=\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}.$$