

173.

**Problem 21.53 (RHK)**

*A particle of mass  $m$  travelling at a relativistic speed makes a completely inelastic collision with an identical particle that is initially at rest. We have to find (a) the speed of the resulting single particle and (b) its mass. We will express our answers in terms of the Lorentz factor  $\gamma$  of the incident particle.*

**Solution:**

(a)

Let the mass of the particles be  $m$ .

Let the speed of the incident particle be  $v$ .

The Lorentz factor  $\gamma$  is defined as  $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$ .

We note  $\gamma v \equiv c(\gamma^2 - 1)^{1/2}$ .

Initial momentum of the two particles one of which is at rest is

$$P_i = \frac{mv}{\sqrt{1 - v^2/c^2}} = m\gamma v = mc(\gamma^2 - 1)^{1/2}.$$

The initial total energy of the two particle system is

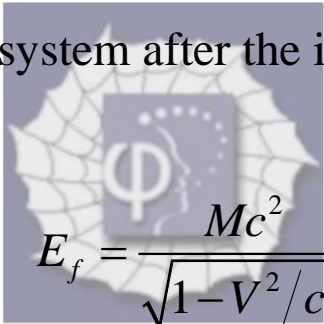
$$E_i = mc^2 + mc^2\gamma = mc^2(1 + \gamma).$$

Let the mass of the two-particle system after the inelastic collision be  $M$  and its speed be  $V$ .

The momentum of the system after the inelastic collision will be

$$P_f = \frac{MV}{\sqrt{1 - V^2/c^2}}.$$

The energy of the system after the inelastic collision will be


$$E_f = \frac{Mc^2}{\sqrt{1 - V^2/c^2}}.$$

From conservation of energy, we have

$$E_f = E_i ,$$

or

$$\frac{Mc^2}{\sqrt{1 - V^2/c^2}} = mc^2(1 + \gamma) ,$$

or

$$\frac{M}{\sqrt{1 - V^2/c^2}} = m(1 + \gamma).$$

From conservation of momentum, we have

$$P_i = P_f ,$$

or

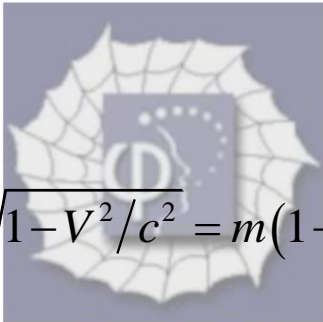
$$\frac{MV}{\sqrt{1-V^2/c^2}} = mc(\gamma^2 - 1)^{1/2} ,$$

or

$$V/c = \frac{(\gamma^2 - 1)^{1/2}}{(\gamma + 1)} = \left( \frac{(\gamma - 1)}{(\gamma + 1)} \right)^{1/2} .$$

(b)

And,


$$\begin{aligned} M &= m(1 + \gamma) \sqrt{1 - V^2/c^2} = m(1 + \gamma) \left( 1 - \frac{(\gamma - 1)}{(\gamma + 1)} \right)^{1/2} , \\ &= m\sqrt{2(\gamma + 1)} . \end{aligned}$$