173.

Problem 21.53 (RHK)

A particle of mass m travelling at a relativistic speed makes a completely inelastic collision with an identical particle that is initially at rest. We have to find (a) *the speed of the resulting single particle and* (b) *its mass. We will express our answers in terms of the Lorentz factor* γ *of the incident particle.*

Solution:

(a)

Let the mass of the particles be *m*.

Let the speed of the incident particle be *v*.

The Lorentz factor γ is defined as $\gamma = \dfrac{1}{\sqrt{1 - v^2/c^2}}$.

We note $\gamma v \equiv c\left(\gamma^2 - 1\right)^{1/2}$.

Initial momentum of the two particles one of which is at rest is

$$P_i = \frac{mv}{\sqrt{1 - v^2/c^2}} = m\gamma v = mc\left(\gamma^2 - 1\right)^{1/2}.$$

The initial total energy of the two particle system is

$E_i = mc^2 + mc^2\gamma = mc^2(1+\gamma).$

Let the mass of the two-particle system after the inelastic collision be M and its speed be V.

The momentum of the system after the inelastic collision will be

$$P_f = \frac{MV}{\sqrt{1-V^2/c^2}}.$$

The energy of the system after the inelastic collision will be

$$E_f = \frac{Mc^2}{\sqrt{1-V^2/c^2}}.$$

From conservation of energy, we have

$$E_f = E_i \ ,$$

or

$$\frac{Mc^2}{\sqrt{1-V^2/c^2}} = mc^2\left(1+\gamma\right) \ ,$$

or

$$\frac{M}{\sqrt{1-V^2/c^2}} = m\left(1+\gamma\right).$$

From conservation of momentum, we have

$$P_i = P_f \ ,$$

or

$$\frac{MV}{\sqrt{1-V^2/c^2}} = mc\left(\gamma^2 - 1\right)^{1/2},$$

or

$$V/c = \frac{\left(\gamma^2 - 1\right)^{1/2}}{(\gamma + 1)} = \left(\frac{(\gamma - 1)}{(\gamma + 1)}\right)^{1/2}.$$

(b)

And,

$$M = m(1+\gamma)\sqrt{1-V^2/c^2} = m(1+\gamma)\left(1 - \frac{(\gamma-1)}{(\gamma+1)}\right)^{1/2},$$

$$= m\sqrt{2(\gamma+1)}.$$