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Problem 21.49 (RHK)

If the kinetic energy K and the momentum p of a particle can be measured, it should be possible to find its mass m and thus identify the particle. (a) *We have to show that*

$$m = \frac{(pc)^2 - K^2}{2Kc^2}.$$

(b) *We have to find the limit of this expression as $v/c \to 0$. We have to find the mass of a particle whose kinetic energy is* 55.0 MeV *and whose momentum is* $121 \text{ MeV } c^{-1}$ *and we have to express our answer in terms of mass m_e of the electron.*

Solution:

We know that for a particle of mass m and momentum p its energy, E, is given by

$$E^2 = (pc)^2 + m^2c^4.$$

The kinetic energy K is by definition

$$K = E - mc^2.$$

We thus have

$$\left(K + mc^2\right)^2 = E^2 \ ,$$

or

$$K^2 + m^2c^4 + 2K\left(mc^2\right) = \left(pc\right)^2 + m^2c^4.$$

From this equation we get

$$m = \frac{\left(pc\right)^2 - K^2}{2Kc^2}.$$

(b)

For obtaining the $\lim\ v/c \to 0$ of the above expression, we will rewrite this expression as

$$m = \frac{\left(p/c\right)^2 - \left(K/c^2\right)^2}{2\,K/c^2}.$$

We use the results that in the $\lim\ \beta = v/c \to 0,$

$$p/c = m\beta\left(1+\tfrac{1}{2}\beta^2 + O\left(\beta^4\right)\right),$$

$$K/c^2 = \tfrac{1}{2}m\beta^2\left(1+\tfrac{1}{4}\beta^2 + O\left(\beta^4\right)\right).$$

$$\lim \beta \to 0 \ \frac{\left(p/c\right)^2 - \left(K/c^2\right)^2}{2K/c^2} = \frac{m^2\beta^2\left(1-\tfrac{3}{4}\beta^2 + O\left(\beta^4\right)\right)}{m\beta^2\left(1+\tfrac{1}{4}\beta^2 + O\left(\beta^4\right)\right)},$$

$$= m.$$

(c)

The mass of the particle, whose kinetic energy,

$K = 55.0$ MeV , and whose momentum,

$p = 121$ MeV c^{-1},will therefore be

$$mc^2 = \frac{(121)^2 - (55)^2}{2\times 55} \text{ MeV}$$

$$= 105.6 \text{ MeV}.$$

As

$m_e c^2 = 0.511$ MeV,

$m = 206.6\ m_e.$