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Problem 21.49 (RHK)

If the kinetic energy K and the momentum p of a particle can be measured, it should be possible to find its mass m and thus identify the particle. (a) We have to show that

$$m = \frac{(pc)^2 - K^2}{2Kc^2}.$$

(b) We have to find the limit of this expression as $v/c \rightarrow 0$. We have to find the mass of a particle whose kinetic energy is 55.0 MeV and whose momentum is $121 \text{ MeV } c^{-1}$ and we have to express our answer in terms of mass m_e of the electron.

Solution:

We know that for a particle of mass m and momentum p its energy, E , is given by

$$E^2 = (pc)^2 + m^2c^4.$$

The kinetic energy K is by definition

$$K = E - mc^2.$$

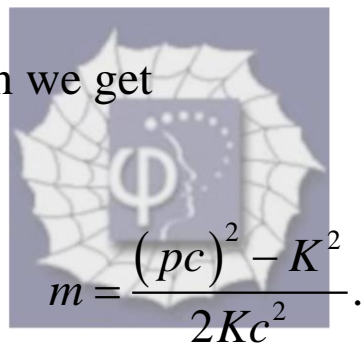
We thus have

$$(K + mc^2)^2 = E^2,$$

or

$$K^2 + m^2c^4 + 2K(mc^2) = (pc)^2 + m^2c^4.$$

From this equation we get


$$m = \frac{(pc)^2 - K^2}{2Kc^2}.$$

(b)

For obtaining the $\lim v/c \rightarrow 0$ of the above expression, we will rewrite this expression as

$$m = \frac{(p/c)^2 - (K/c^2)^2}{2K/c^2}.$$

We use the results that in the $\lim \beta = v/c \rightarrow 0$,

$$p/c = m\beta\left(1 + \frac{1}{2}\beta^2 + O(\beta^4)\right),$$

$$K/c^2 = \frac{1}{2}m\beta^2\left(1 + \frac{1}{4}\beta^2 + O(\beta^4)\right).$$

$$\lim_{\beta \rightarrow 0} \frac{(p/c)^2 - (K/c^2)^2}{2K/c^2} = \frac{m^2\beta^2\left(1 - \frac{3}{4}\beta^2 + O(\beta^4)\right)}{m\beta^2\left(1 + \frac{1}{4}\beta^2 + O(\beta^4)\right)},$$

$$= m.$$

(c)

The mass of the particle, whose kinetic energy,

$K = 55.0 \text{ MeV}$, and whose momentum,

$p = 121 \text{ MeV } c^{-1}$, will therefore be

$$mc^2 = \frac{(121)^2 - (55)^2}{2 \times 55} \text{ MeV}$$

$$= 105.6 \text{ MeV}.$$

As

$$m_e c^2 = 0.511 \text{ MeV},$$

$$m = 206.6 m_e.$$