170.

Problem 21.35 (RHK)

Observers S and S' *stand at the origin of their respective frames, which are moving relative to each other with a speed* 0.600 c. *Each has a standard clock, which, as usual they set to zero when the two origins coincide. Observer S keeps the* S' *clock visually in sight. We have to find* (a) *the time the* S' *clock will record when the S clock records* $5.0\ \mu\text{s}$. (b) *We have the find the time that observer* S *will actually read on the* S' *clock when the S clock reads* $5.0\ \mu\text{s}$.

Solution:

(a)

We will use the Lorentz transformations connecting the space-time co-ordinates of the same event observed by the observers in the frame S and that in the frame S'.

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}},$$

$$t' = \frac{t - xv/c^2}{\sqrt{1 - v^2/c^2}}.$$

When the clock of S shows $5.0\ \mu\text{s}$ the co-ordinates of the origin of S' as measured by S will be

$$x = 5.0 \times 10^{-6} v \text{ m.}$$

and

$$t = 5.0 \times 10^{-6} \text{ s.}$$

The speed of the frame S', with respect to S is $v = 0.6\ c$.

The time t' shown on the clock of S' for this event will be

$$t' = \frac{t - xv/c^2}{\sqrt{1 - v^2/c^2}} = \frac{5.0 \times 10^{-6} - 5.0 \times 10^{-6} v^2/c^2}{\sqrt{1 - v^2/c^2}} \text{s},$$

$$= 5.0 \times 10^{-6} \times \sqrt{1 - 0.6^2} \text{ s} = 4.0 \mu\text{s}.$$

(b)

At the instant when the S-clock reads $5.0\ \mu\text{s}$ the signal communicating the reading on the clock of S' would have left it when it was at a distance l from S such that

$$l = \left(5.0 \times 10^{-6} - l/c\right) \times v,$$

as $v = 0.6\ c$,

$$l = \frac{5.0 \times 10^{-6} \times 0.6\ c}{1 + 0.6} = \frac{3.0 \times 10^{-6} c}{1.6}.$$

The co-ordinates of this event in the S frame are

$$x = \frac{3.0 \times 10^{-6} c}{1.6},$$
$$t = (5.0 - 3.0/1.6) \times 10^{-6} \text{s}.$$

By Lorentz transformation we will calculate t' for this event. It is

$$t' = \frac{(5.0 - 3.0/1.6) - 3.0 \times 0.6/1.6}{0.8} \times 10^{-6} \text{s},$$
$$= 2.5 \mu\text{s}.$$

The observer on S will read $2.5\ \mu\text{s}$ on the clock of S' when its clock shows $5.0\ \mu\text{s}$.