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Problem 21.23 (RHK)

A spaceship, at rest in a certain reference frame S, is given a speed increment of 0.500 *c. It is then given a further* 0.500 *c increment in this new frame. This process is continued until its speed with respect to the original frame S exceeds* 0.999 *c. We have to find the number of increments that are required.*

Solution:

We will solve this problem by using relativistic velocity addition successively. When the spaceship has been given the first speed increment of 0.500 *c* the speed of its rest frame S^1 with respect to *S* will be 0.500 *c*. In S^1 the spaceship is given a speed increment of 0.500 *c*. Let the rest frame of the spaceship be now S^2. Using velocity addition we calculate the speed of S^2 with respect to *S*. It will be

$$\frac{(0.5+0.5)c}{1+0.5^2} = 0.8\ c.$$

In the frame S^2 the spaceship is given a speed increment of 0.500 *c*. Let the new rest frame of the spaceship be called S^3. The speed of S^3 with respect to *S* will be

$$\frac{(0.8+0.5)c}{1+0.8\times 0.5} = 0.928\ c.$$

In the frame S^3 the spaceship is given a speed increment of 0.500 *c*. Let the new rest frame of the spaceship be called S^4. The speed of S^4 with respect to *S* will be

$$\frac{(0.928+0.5)c}{1+0.928\times 0.5} = 0.975\ c.$$

In the frame S^4 the spaceship is given a speed increment of 0.500 *c*. Let the new rest frame of the spaceship be called S^5. The speed of S^5 with respect to *S* will be

$$\frac{(0.975+0.5)c}{1+0.975\times 0.5} = 0.992\ c.$$

In the frame S^5 the spaceship is given a speed increment of 0.500 *c*. Let the new rest frame of the spaceship be called S^6. The speed of S^6 with respect to *S* will be

$$\frac{(0.992+0.5)c}{1+0.992\times 0.5}=0.997\ c.$$

In the frame S^6 the spaceship is given a speed increment of 0.500 *c*. Let the new rest frame of the spaceship be called S^7. The speed of S^7 with respect to *S* will be

$$\frac{(0.997+0.5)c}{1+0.997\times 0.5}=0.999\ c.$$

Thus we find that seven increments of 0.5 *c* are required to enable a spaceship at rest to attain speed of 0.999 *c*.