

168.

**Problem 21.23 (RHK)**

*A spaceship, at rest in a certain reference frame  $S$ , is given a speed increment of  $0.500 c$ . It is then given a further  $0.500 c$  increment in this new frame. This process is continued until its speed with respect to the original frame  $S$  exceeds  $0.999 c$ . We have to find the number of increments that are required.*

**Solution:**

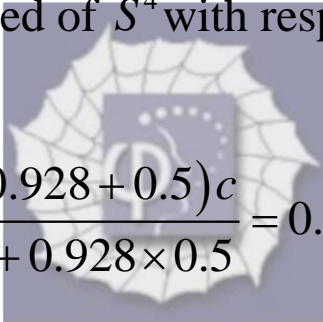
We will solve this problem by using relativistic velocity addition successively. When the spaceship has been given the first speed increment of  $0.500 c$  the speed of its rest frame  $S^1$  with respect to  $S$  will be  $0.500 c$ . In  $S^1$  the spaceship is given a speed increment of  $0.500 c$ . Let the rest frame of the spaceship be now  $S^2$ . Using velocity addition we calculate the speed of  $S^2$  with respect to  $S$ . It will be

$$\frac{(0.5 + 0.5)c}{1 + 0.5^2} = 0.8 c.$$

In the frame  $S^2$  the spaceship is given a speed increment of  $0.500 c$ . Let the new rest frame of the spaceship be called  $S^3$ . The speed of  $S^3$  with respect to  $S$  will be

$$\frac{(0.8 + 0.5)c}{1 + 0.8 \times 0.5} = 0.928 c.$$

In the frame  $S^3$  the spaceship is given a speed increment of  $0.500 c$ . Let the new rest frame of the spaceship be called  $S^4$ . The speed of  $S^4$  with respect to  $S$  will be


$$\frac{(0.928 + 0.5)c}{1 + 0.928 \times 0.5} = 0.975 c.$$

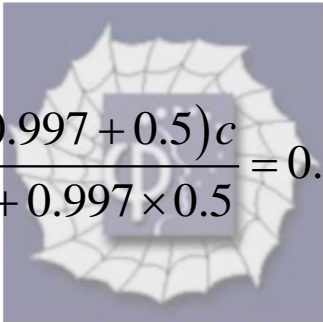
In the frame  $S^4$  the spaceship is given a speed increment of  $0.500 c$ . Let the new rest frame of the spaceship be called  $S^5$ . The speed of  $S^5$  with respect to  $S$  will be

$$\frac{(0.975 + 0.5)c}{1 + 0.975 \times 0.5} = 0.992 c.$$

In the frame  $S^5$  the spaceship is given a speed increment of  $0.500 c$ . Let the new rest frame of the spaceship be called  $S^6$ . The speed of  $S^6$  with respect to  $S$  will be

$$\frac{(0.992 + 0.5)c}{1 + 0.992 \times 0.5} = 0.997 c.$$

In the frame  $S^6$  the spaceship is given a speed increment of  $0.500 c$ . Let the new rest frame of the spaceship be called  $S^7$ . The speed of  $S^7$  with respect to  $S$  will be


$$\frac{(0.997 + 0.5)c}{1 + 0.997 \times 0.5} = 0.999 c.$$

Thus we find that seven increments of  $0.5 c$  are required to enable a spaceship at rest to attain speed of  $0.999 c$ .